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INDUCED SHAPE FIBRATIONS AND FIBER SHAPE EQUIVALENCE

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ABSTRACT. In this paper we prove that a map induced from a shape fibration is a shape fibration. We define a fiber shape equivalence relation between shape fibrations. Also, generalizing the homotopy relation, we define a strong equivalence relation in the set of maps between compact metric spaces. Then we prove that two strongly equivalent maps induce fiber shape equivalent shape fibrations. As a corollary we show that the fibers over two points connected by a strong shape path are of the same shape. Finally, we prove that a fiber shape equivalence induces a relative shape map which induces an appropriate isomorphism on relative shape groups.

1. Introduction. In a recent paper [**11**] S. Mardešić and T.B. Rushing have defined an important notion of 'shape fibration' by generalizing an approximate fibration of Coram and Duvall [**4**]. One expands a map $p: E \to B$ between compact metric spaces into a map $\mathbf{p}: \mathbf{E} = (E_n, r_{nm}) \to \mathbf{B} = (B_n, q_{nm})$ of inverse sequences of compact ANR's. The map p is a shape fibration if $\mathbf{p}$ has the following approximate homotopy lifting property: each n and each $\varepsilon > 0$ admit an index $m \geqq n$ and $\delta > 0$ such that for any topological space X, whenever the maps $h: X \to E_m$ and $H: X \times I \to B_m$ satisfy $d(p_m h, H_0) \leqq \delta$, then there is an homotopy $G: X \times I \to E_n$ satisfying $d(G_0, r_{nm}h) < \varepsilon$ and $d(p_n G, q_{nm}H) < \varepsilon$.

Analogously to fibrations, one may ask the following questions for shape fibrations. Is a map induced from a shape fibration a shape fibration? Is there a notion of a fiber shape map? In what sense are two shape fibrations fiber shape equivalent? Is it true that two homotopic maps induce equivalent shape fibrations?

In this paper we have studied all these questions and have found satisfactory positive answers. §2 contains basic definitions and some basic results that we need. In §3 we prove that a map induced from a shape fibration is a shape fibration. §4 contains the definition of a fiber shape equivalence.

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