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# ALMOST PERIODIC FUNCTIONS ON SEMITOPOLOGICAL SEMIGROUPS

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Let $S$ be a semitopological semigroup with identity $e$. One of the ways of defining almost periodicity of a function $f \in C(S)$ is to say that, (*) for each $\varepsilon > 0$, there is a finite subset $A \subset S$ such that the set of left translates $\{L_s f \mid s \in S\}$ is contained in $\{h \in C(S) \mid \|L_s f - h\| < \varepsilon$ for some $s \in A\}$. By allowing the subset $A$ to be totally bounded (suitably defined) or compact in this and related definitions, one obtains a large number of function classes. Recently $T$. Kayano defined some of these classes (none involving $A$ compact) and gave some relationships among them. In the present note, we tighten up some of his results and show in particular that $f \in C(S)$ is almost periodic if $f$ satisfies the condition (*) above with totally bounded sets $A$ and also $\|L_s f - f\| \to 0$ whenever $s \to e$. We also present some examples and a theorem showing some classes can be different; for example, functions satisfying (*) with sets $A$ compact or totally bounded need not be almost periodic.

**Preliminaries and first results.** A semigroup $S$ with identity $e$ that is also a topological space is called a *semitopological semigroup* if the maps $s \to st$ and $s \to ts$ are continuous from $S$ into $S$ for all $t \in S$; if, as well, $S$ admits inverses, i.e., if $S$ is a group, it is called a *semitopological group*. We denote by $C(S)$ the space of bounded continuous complex-valued functions on $S$ furnished with the supremum norm $\|f\| = \sup_{s \in S}|f(s)|$. The *left translate* $L_t f$ of $f \in C(S)$ by $t \in S$ is defined by $L_t f(s) = f(ts)$ for all $s \in S$, and a subset $A$ of $S$ is called *right* [*left*] *totally bounded* if, given any neighbourhood $V$ of $e$, there exist a natural number $N$ and $t_1, \ldots, t_N \in A$ such that

$$A \subset \bigcup \{Vt_i \mid 1 \leqq i \leqq n\} \; [A \subset \bigcup \{t_i V \mid 1 \leqq i \leqq N\}].$$

We now wish to define some classes of functions in $C(S)$, classes involving a family $\mathscr{A}$ of subsets of $S$. We shall say $f \in C(S)$ satisfies conditions

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