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ENDOMORPHISM RINGS AND SUBGROUPS OF FINITE RANK TORSION-FREE ABELIAN GROUPS

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Let A be a finite rank torsion-free abelian group and let $E(A)$ denote the endomorphism ring of A. Then $Q \otimes_Z E(A) = QE(A)$ and $E(A)/pE(A)$ are artinian algebras, where Z is the ring of integers, Q is the field of rationals, and p is a prime of Z.

Define A to be *Q-simple* if $QE(A)$ is a simple algebra, and *p-simple* for a prime p of Z if $pE(A) = E(A)$ or if $E(A)/pE(A)$ is a simple algebra. In contrast to finite rank torsion-free groups in general, groups that are p-simple for each p have some pleasant decomposition properties.

THEOREM I. *A reduced group A is p-simple for each prime p of Z if and only if $A = A_1 \oplus \cdots \oplus A_k$, where each A_i is fully invariant in A, each A_i is Q-simple and p-simple for each prime p of Z, and if p is a prime of Z then there is some j with $A/pA = A_j/pA_j$.*

THEOREM II. *A group A is Q-simple and p-simple for each prime p of Z if and only if $A = B_1 \oplus \cdots \oplus B_n$, where each B_i is strongly indecomposable, Q-simple and p-simple for each prime p of Z and B_i is nearly isomorphic to B_j (in the sense of Lady* [7]) *for each i and j.*

Suppose that A is Q-simple and p-simple for each prime p of Z. Then A is indecomposable if and only if A is strongly indecomposable. Furthermore, if $S =$ Center $E(A)$, then S is a subring of an algebraic number field such that every element of S is a rational integral multiple of a unit of S, as described in **[1]**, and $E(A)$ is a maximal S-order in $QE(A)$.

Examples of groups that are Q-simple and p-simple for each prime p of Z include: indecomposable strongly homogeneous groups (characterized in **[1]**); indecomposable groups with p-rank $\leqq 1$ for each prime p of Z (Murley **[8]**); and indecomposable quasi-pure-projective and quasi-pure-injective groups (**[4]**).

Define A to be *irreducible* if QA is an irreducible $QE(A)$-module (Reid **[10]**) and *p-irreducible*, for a prime p of Z, if A/pA is an irreducible $E(A)/pE(A)$-module. If A is irreducible (p-irreducible), then A is Q-simple

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