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BOUNDS FOR VISCOSITY PROFILES FOR 2 × 2 SYSTEMS OF CONSERVATION LAWS

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ABSTRACT. Given a system of two conservation laws which is admissible and satisfies the half-plane condition introduced by Keyfitz and Kranzer, the existence of a unique travelling-wave solution of the associated parabolic system, $U_t + F(U)_x = \varepsilon U_{xx}$, which approximates a given shock, is proved. The shock profile trajectory is a convex curve in phase space, bounded by forward and backward shock curves.

1. Introduction. In [7], an existence theorem for solutions to the Riemann problem was proved for a class of genuinely nonlinear 2 × 2 conservation laws that is somewhat larger than those previously considered (see [9] and the references in [7]). In this article, we show that viscosity shock profiles in the form of travelling wave solutions to the associated parabolic system

$$U_t + F(U)_x = \varepsilon U_{xx} \tag{1}$$

can be constructed for this same class of equations. This enlarges the class considered by Conley and Smoller in [2] and does away with the need for any additional assumptions such as appear in their paper. We also obtain more satisfactory bounds on the trajectories of the travelling wave solutions, and show that the trajectories are convex curves.

One consequence of this result is that the shock wave solutions of

$$U_t + F(U)_x = 0 \tag{2}$$

for the class of equations considered in [7] are the limits of solutions of (1) as ε tends to zero. This verifies the admissibility condition of Gel'fand [5] for this larger class and without additional assumptions. There are also implications for the construction of solutions to the Cauchy problem for (1) by a vanishing viscosity method, although formidable difficulties remain in carrying out this procedure. The bounds obtained here may be useful.

We begin with some background. In (2), $U = (u_1, u_2)$ is a function of x

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