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PERTURBATIONS OF A BOUNDARY VALUE PROBLEM WITH POSITIVE, INCREASING AND CONVEX NONLINEARITY

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1. Introduction. Let ρ_t be a family of positive functions:

$$\rho_t(x) = \rho_0(x) + t\pi(x), \ x \in [-1, +1], \ t \in [-1, +1].$$

For a fixed t we consider the boundary value problem (BVP):

$$(\mathrm{BVP}t)\begin{cases} -u''(x) = \lambda \rho_t(x) f(u(x)), \ x \in (-1, +1) \\ u(-1) \ = u(+1) = 0, \end{cases}$$

where λ is a non-negative parameter and f a positive, increasing and convex function. Under these conditions there is a critical value $\lambda_t^* > 0$ such that (BVPt) has at least one solution for $\lambda \in (0, \lambda_t^*)$ and no solution for $\lambda > \lambda_t^*$.

Thinking of (BVP$t = 0$) as the unperturbed problem, it is the purpose of this paper to study λ_t^* as a function of the perturbation parameter t. Our result is a condition which implies the inequality $\lambda_t^* < \lambda_0^*$ for small positive (or negative) t. This condition involves only the perturbation π and the solutions of (BVP0) at λ_0^* and of its linearization. The method which leads to this result is to develop (BVPt) around the unperturbed problem. Thus we find a bifurcation equation in t, which has to be discussed.

Our paper is organized as follows: §2 hypotheses; §3 here we reproduce some known results which we use in the next section; §4 statement and proof of our perturbation lemma.

2. Hypotheses. Let $I = \{x \in \mathbf{R}/|x| < 1\}$, $\bar{I}$ its closure, $\mathbf{R}_+ = \{\xi \in \mathbf{R}/ \xi \geqq 0\}$, $\lambda \in R_+$. We make the following hypotheses:

H1) $\rho_0; \bar{I} \to \mathbf{R}$ continuous and positive.
π: $\bar{I} \to \mathbf{R}$ continuous and $|\pi(x)| < \rho_0(x)$, $x \in \bar{I}$.
$\rho_t(x) = \rho_0(x), + t\pi(x)$, $x \in \bar{I}$, $t \in \bar{I}$.

H2) f: $\mathbf{R}_+ \to \mathbf{R}$ continuously differentiable and

$$f(0) > 0, \lim_{\xi \to +\infty} \frac{f(\xi)}{\xi} = \infty, f'(0) \geqq 0, f' \text{ strictly increasing.}$$

Thus f is positive, strictly increasing and strictly convex. We write

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