ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 12, Number 2, Spring 1982

ON THE EXISTENCE OF UNIQUE EIGENSETS OF MONOTONE PROCESSES

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ABSTRACT. A Sufficient condition is given to guarantee the existence of a unique eigenset of a monotone process. Then, a special class of monotone processes is proved to have unique eigensets through this condition and the Perron-Frobenius Theorem.

1. Introduction. Rockafellar [**4**, p. 69, Theorem 4] proved a theorem which provides necessary and sufficient conditions for the existence of unique eigensets of monotone processes. Since those necessary and sufficient conditions must be satisfied by every pair of non-singular monotone sets in P_n and P_n^*, it is almost impossible to verify that a certain monotone process actually satisfies these conditions. In this paper, a sufficient condition in a simpler form is given to guarantee the existence of a unique eigenset. This sufficient condition in fact is a modification of Rockafellar's conditions. Then, a special class of monotone processes is proved to have unique eigensets through this modified condition and the Perron-Frobenius Theorem [**2**].

We shall only give the definitions of monotone sets, monotone processes, and eigensets of a monotone process. For more detailed definitions (e.g., positively homogeneous, sub-additive, non-singular, etc.), examples, and properties of monotone processes see [**3**], [**4**], and the references therein.

DEFINITION 1.1. [**4**, p. 11]. A monotone set of concave type in P_n, the nonnegative orthant of R^n, is a non-empty closed bounded convex set C such that $0 \leqq y_1 \leqq y_2 \in C$ implies $y_1 \in C$. A monotone set of convex type is a non-empty closed convex set such that $y_1 \geqq y_2 \in C$ implies $y_1 \in C$.

DEFINITION 1.2. [**4**, p. 9]. A monotone process of concave type from P_n to P_m is a nonnegative process T which is positively homogeneous, sub-additive, closed, and satisfies

(a) $T(x)$ is a monotone set of concave type for all $x \in P_n$, and

(b) $0 \leqq x_1 \leqq x_2$ implies $T(x_1) \subseteqq T(x_2)$.

AMS (*MOS*) *subject classification* (1970): Primary 15A18, 90A99.
*Sponsored by the University of Alabama under Project Number 951.
Received by the editors on March 25, 1980, and in revised form on Dec. 20, 1980.