

MODERN PERSPECTIVES ON CLASSICAL FUNCTION THEORY

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Not long ago, I came across an article by a famous French mathematician, one of the foremost exponents of the Bourbakist school of mathematics, purporting to describe the present direction of mathematical research. By the end of the third paragraph he had managed to dismiss the theory of analytic functions of a complex variable as having cut itself off from the "main stream" of mathematics by "indulging in overly specialized questions." Well, what can one reasonably expect from someone who asserts that "the invention of functors is one of the main goals of modern mathematicians"? Perhaps benign neglect by those who favor the Grand Design is not such a bad thing; it enables complex analysts to work in peace. Not yet, at least, has anyone laid function theory on the Procrustean bed of his own ideology and tried to trim it, head, limbs, and all, to the specifications of his own taste, whim, or fancy. If function theory is to be dubbed a "living fossil" (like the Jews, in Toynbee's scheme of history), so be it.

Actually, the situation is not so bad. A discipline which can boast contemporary exponents of the caliber of Nevanlinna, Ahlfors, Beurling, and Schiffer (not to mention the bright stars of several younger generations) is surely far from played out. To tell the truth, few sensible people ever thought it was. I had to look long and hard for an unfavorable comment, and in the process I encountered numerous unsolicited encomia from men of such high sensibility and diverse interests as Eugene Wigner, Felix Browder, Georg Kreisel, and Clifford Truesdell (references available on request). For such individuals, impervious to the fad of the hour, complex variables has a permanent value.

And yet, there is something in Professor Dieudonné's assessment [4] that strikes a nerve. Function theory is a little bit like Euclid. All of us have had to learn some, and the basic theory is so coherent, so all of a piece, hangs together so well with no loose ends, that there is the ever present temptation to conclude that one has learned it all.

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