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THE HULLS OF C(Y)

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Introduction. Let $C(Y)$ be the set of all continuous real-valued functions on a completely regular space Y. Then $C(Y)$ can be considered as an ℓ-group G_1 or as a semiprime ring G_3, and in each case it admits various X-hulls, which are minimal essential extensions with some property X. We show that G_1^X is essentially the same as G_3^X and investigate the structure of these X-hulls. All of these hulls are contained in the complete ring of quotients $Q(Y)$ of G_3, and, in fact, $Q(Y)$ is the lateral completion of G_1 or of G_3.

In the first two sections we summarize the theory known for abelian ℓ-group and commutative semiprime ring X-hulls. The third section contains a description of the hulls of $C(Y)$, and their relationships with one another. §4 contains characterizations of $C(Y)$ considered as an abstract ℓ-group.

For further information about lattice-ordered groups (ℓ-groups), see **[9]** or **[14]**; for semiprime rings, see **[26]**; for $C(Y)$, see **[24]**.

We will use $\sum T_\lambda (\Pi T_\lambda)$ to represent the restricted (unrestricted) direct product of the groups or rings T_λ; in the case of ℓ-groups, these groups are equipped with the cardinal order.

We wish to acknowledge the valuable advice of Jack Porter about the topological results that appear in this paper. In particular, Theorem 3.9 and Example 3.12 are entirely due to him.

1. The hulls of semiprime rings. Throughout this section let G be a commutative semiprime ring (that is, G is a subdirect product of integral domains) with identity. We summarize some of the X-hull theory of G that is developed in **[18]**, **[19]**, and **[20]**. Actually, this theory also holds for non-commutative semiprime rings.

For $a, b \in G$ define $a \underset{\sim}{\alpha} b$ if $a^2 = ab$. This is a partial order for G (introduced in **[1]**) with smallest element 0 and for $a, b, x \in G$, $a \underset{\sim}{\alpha} b$ implies that $ax \underset{\sim}{\alpha} bx$. Moreover, $a \underset{\sim}{\alpha} b$ if and only if in each representation of $G \subseteqq \Pi T_\lambda$ as a subdirect product of integral domains T_λ, $a_\lambda \neq 0$ implies that $a_\lambda = b_\lambda$.

One says that a is *disjoint* from b or that a is *orthogonal* to b if $ab = 0$ (notation: $a \perp b$). This is equivalent to the fact that a and b have disjoint

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