

## EXTENSIONS OF MAPS DEFINED ON CONVERGENCE SPACES

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**ABSTRACT.** This is a foundational study of the extendibility of various continuous type maps defined on a dense subspace  $X$  of a preconvergence space  $Z$ . A minimal property (weak-admissibility) for such extensions is established and is applied for the case where the remainder  $R = Z - X$  is  $U$ -principal. Major results include necessary and sufficient conditions for the extendibility of continuous [resp. weakly-continuous] mappings, a general Taimanov type characterization for extendibility and a general result which shows that a weakly-admissible map defined on  $X$  can be extended to a weak- $n$ -continuous map on  $Z$  where  $Z$  is any extension of  $X$ . Finally, numerous examples are given which show that the major results obtained are nontrivial and have many well-known propositions as corollaries.

**1. Introduction.** In their paper [24], the Steiners make the following remark relative to topological spaces, "Our point of view is that one of the most important kinds of information a structure on a space  $X$  provides, besides a topology for  $X$ , is a topological extension of  $X$ ." The Steiners' philosophy can obviously be applied to the more general concept of the convergence structure on a set  $X$  as introduced by D. Kent in his foundational papers [10], [11] and [12].

Various types of extensions for a convergence space  $X$  have been recently investigated. The majority of these extensions are compactifications of one type or another (see [18], [19], [20], [21] and [27]). Once these extensions have been obtained, then, as in the case for topological spaces, the most useful investigation appears to be in the general area of the extendibility of continuous maps onto these space extensions. Indeed, with respect to the Steiners' paper [24], D. F. Wooten initiated such a research project for semi-uniform spaces in his paper [28]. Other results relative to the extensions of maps on semi-uniform spaces can also be found in [6].

The major goal of this present investigation is to add to what is an apparent void in the theory of extensions for maps on convergence spaces. This present research is a foundational study of such concepts, where the maps we wish to extend have a minimal property necessary for extendibil-

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