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BOREL EXCEPTIONAL VALUES IN THE UNIT DISK

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1. Introduction. Some time ago E. Borel [**1**] introduced for entire functions the idea of a Borel exceptional value. The analogous idea for functions meromorphic in the unit disk **D** has been considered by R. Nevanlinna [**4**, p. 144] (indirectly) and by M. Tsuji [**6**, p. 293]. Below we extend some of Tsuji's results and consider analogues of results of G. Valiron [**7**, p. 71–78] and S. Singh and H. Gopalakrishna [**5**] as well as some relations between Borel exceptional values and other types of exceptional values studied in value distribution theory. Contrary to the situation for entire functions, an analytic function in the unit disk may have a Borel exceptional value and not have regular growth. We shall use the notation of Nevanlinna theory (see, for example, W. Hayman [**3**]).

For our purposes we define the *order* ρ of a meromorphic function f defined in **D** by

$$\rho = \limsup_{r \to 1} \frac{\log^+ T(r, f)}{-\log(1-r)},$$

and the *lower order* λ by

$$\lambda = \liminf_{r \to 1} \frac{\log^+ T(r, f)}{-\log(1-r)}.$$

Let $\{a_n\}$ be the zeros of $f(z) - a$ for z in **D**. Define the *convergence exponent* $\mu_a \geqq 0$ of $\{a_n\}$ as follows:

(i) If $\sum_n (1 - |a_n|) < \infty$, then $\mu_a = 0$.

(ii) If $\sum_n (1 - |a_n|) = \infty$, then $\mu_a = \mu$ is that number such that for any $\varepsilon > 0$

$$\sum_n (1 - |a_n|)^{\mu+1-\varepsilon} = \infty \text{ and } \sum_n (1 - |a_n|)^{\mu+1+\varepsilon} < \infty.$$

Tsuji [**6**, p. 204] notes that

$$\int_{r_0}^1 N(r, a)(1-r)^{\lambda-1} dr \text{ and } \sum_n (1 - |a_n|)^{\lambda+1}, \lambda > 0,$$

converge or diverge simultaneously. So

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