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THE MAXIMAL RING OF QUOTIENTS OF A FINITE VON NEUMANN ALGEBRA

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ABSTRACT. Expository article on the embedding of finite von Neumann algebras into regular rings. The algebra of affiliated closed operators is identified with the maximal ring of right quotients. Applications of the theory of self-injective regular rings to operator algebras (matrix algebras, projection lattices, reduction theory).

1. Introduction. Let A be a von Neumann algebra of finite class [**4**, Ch. III, §8]. In their 1936 paper [**17**], F. J. Murray and J. von Neumann showed that A can be embedded in a regular ring $\mathscr{U}(A)$ of closed, densely defined operators "affiliated" with A (in a sense made precise below). Two decades later, Y. Utumi gave a general construction that embeds A in a regular ring Q, which he called its maximal ring of right quotients [**22**]. Twelve years later, J. E. Roos demonstrated that $\mathscr{U}(A)$ and Q are the same ring [**20**]. This apparently glacial progress is due less to the difficulty of the subject than it is to the nearly perfect insulation separating ring theorists from operator theorists. Since the time of von Neumann, and even since the time of Roos' paper, the theory of regular rings has ripened significantly, the maturity of the subject being evident from K. R. Goodearl's recent monograph [**8**]; the regular rings of operator theory can now be perceived in a very general light, and their algebraic properties proved neatly and efficiently. Moreover, these rings are seen, via the general theory of regular rings, to possess a striking property not foreseen from the perspective of operator theory, namely, self-injectivity. This seems, therefore, to be a propitious time to review the regular rings of operator theory, taking advantage of the economies made possible by the algebraic theory of regular rings. The present article is written from the perspective of operator theory, no knowledge of AW^*-algebras being required for the main results; however, the arguments are sufficiently general to apply to AW^*-algebras, yielding, in particular, a brief new proof that the algebra of $n \times n$ matrices over an AW^*-algebra is also an AW^*-algebra. For a purely algebraic view of the subject, the reader should consult the books of Kaplansky and Goodearl ([**15**], [**8**]) and the paper of D. Handelman [**12**].

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