ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 12, Number 1, Winter 1982

MINIMAL H^P INTERPOLATION IN THE CARATHEODORY CLASS

JOSEPH A. CIMA

ABSTRACT. For $C = (c_1, c_2, \cdots, c_n)$ a vector in $\mathbf{C}^n$, let $C(c_1, \ldots, c_n)$ denote the class of analytic functions with Taylor expansion

$$f(z) = 1 + c_1z + \cdots + c_nz^n + \sum_{k=n+1}^{\infty} a_kz^k$$

and $\operatorname{Re} f(z) > 0$ in the unit disc. It is shown that for p fixed in $[1, \infty)$ there is a unique function of least H^p-norm in $C(c_1, \ldots, c_n)$.

1. Introduction. In this paper we give a new and shorter proof a result of Beller and Pinchuk [1] and extend their result to the general case of H^p, $1 \leqq p < \infty$. We consider a minimal interpolation problem at the origin of the unit disc D for the class $H^p \cap C$. H^p is the usual Hardy space of functions analytic in D with p-th integral means bounded. The class C is the Caratheodory class of functions

$$f(z) = 1 + c_1z + c_2z^2 + \cdots$$

analytic in D with $\operatorname{Re} f(z) > 0$ in D. If n complex numbers $c_1, \ldots, c_n$ are given, we wish to prove that there is a unique function f in $H^p \cap C$ of the form

$$f(z) = 1 + c_1z + \cdots + c_nz^n + \sum_{k=n+1}^{\infty} a_kz^k$$

where $\|f\|_p$ is minimal among such functions.

It is well known that the mapping ν_n of C into $\mathbf{C}^n$ by $\nu_n : f \to (c_1, \ldots, c_n)$ has range C_n, which is a compact convex subset of C^n. The following result of C. Caratheodory and 0. Toeplitz appears in [4].

THEOREM. *To each point of* $(C_n)^0$ = *interior* C_n *there correspond infinitely many functions in C. Each boundary point of* C_n *corresponds to only one f in C. The preimages of boundary points are functions of the form*

$$f(z) = \sum_{k=1}^{m} \mu_k\left[\frac{1 + \alpha_kz}{1 - \alpha_kz}\right], \tag{1.1}$$

where $1 \leqq m \leqq n$; $|\alpha_k| = 1$, $\mu_k > 0$ *and* $\sum_{k=1}^{m} \mu_k = 1$.

Received by the editors on December 5, 1979, and in revised form on April 21, 1980.