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CONVERGENCE QUESTIONS FOR LIMIT PERIODIC CONTINUED FRACTIONS

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1. Introduction. Given two sequences of complex numbers $\{a_n\}$, $\{b_n\}$, $n \geqq 1$, we define for complex w

$$s_n(w) = \frac{a_n}{b_n + w}, \; n \geqq 1,$$

and

$$S_N^{(n)}(w) = s_{n+1}\,(S_N^{(n+1)}(w)), \; 0 \leqq n \leqq N-1,$$
$$S_N^{(N)}(w) = w.$$

Using one of the standard notations we then have

$$S_N^{(n)}(w) = \frac{a_{n+1}}{b_{n+1}} + \frac{a_{n+2}}{b_{n+2}} + \cdots + \frac{a_N}{b_N + w}, \; 0 \leqq n \leqq N-1.$$

Instead of $S_N^{(0)}(w)$ we shall usually write $S_N(w)$.

The *continued fraction*

$$\mathrm{K}_{n=1}^{\infty}\left(\frac{a_n}{b_n}\right) = \frac{a_1}{b_1} + \frac{a_2}{b_2} + \cdots + \frac{a_n}{b_n} + \cdots \tag{1.1}$$

then is the ordered pair $\langle\langle\{a_n\}, \{b_n\}\rangle, \{S_n(0)\}\rangle$. Here it is understood that $\{a_n\}$ and $\{b_n\}$ be such that $S_n(0)$ is defined as an extended complex number for all n (or at least from a certain $n = n_0$ on). This is in particular the case if $a_n \neq 0$ for all n or if $a_n = 0$ for all n and simultaneously $b_n \neq 0$. The sequences $\{a_n\}$ and $\{b_n\}$ are called the *sequences of elements*, and $\{S_n(0)\}$ is the *sequence of approximants*. Convergence of a continued fraction means convergence of the sequence of approximants (possibly to ∞). In case of convergence the notation $\mathrm{K}_{n=1}^{\infty}(a_n/b_n)$ is also used for $\lim_{n\to\infty} S_n(0)$.

The approximants of a continued fraction can also be represented as $S_n(0) = A_n/B_n$, where

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