

HEIGHT ONE SEPARABLE ALGEBRAS OVER COMMUTATIVE RINGS

LAWRENCE W. NAYLOR

ABSTRACT. In this paper we define an R -algebra S to be height one separable over R (a commutative ring) if S is separable at each localization at a height one prime ideal of R . We prove some general properties of height one separability and give some examples of non-separable, height one separable extensions. It is also shown that if S is an integrally closed domain and R is the fixed subring of G -invariant elements of S , for some finite group G of automorphisms of S , and if each localization of R at a height 1 prime ideal in R is Noetherian, then S is a height one Galois extension (i.e., each localization at a height one prime ideal of R yields a Galois extension) if and only if S is unramified at each minimal prime ideal in S .

Introduction. In [2], Auslander and Buchsbaum characterize separability for a Noetherian ring S over a base ring R in terms of ramification of prime ideals in S . They prove that, with rather general assumptions, S is R -separable if and only if each maximal ideal of S is unramified. If more conditions are put on R and S , namely that R be an integrally closed Noetherian domain and S the integral closure of R in a separable field extension of the quotient field of R , with S projective as an R -module, they achieve the following result: S is R -separable if and only if each prime ideal of height 1 in S is unramified. We will give examples here to show that this result can fail to hold if the Noetherian restriction on R is removed or if S is not R -projective. The setting here is rather closely related to the problem of the purity of the branch locus (see [1]). One of the examples here will show that if the base ring R is a local ring which is not regular, then purity may indeed fail to hold for R .

We will focus our attention here on the prime ideals of the base ring R , and call S *height 1 separable* over R if S is separable at each localization at a height 1 prime ideal of R . We establish some general properties of height 1 separable algebras and give several examples of height 1 separable algebras which are not separable. In §3 we examine the situation where

AMS 1970 Classification: 13B99.

Key words and phrases: separable extension, Galois extension, height one prime ideal, unramified.

Received by the editors on April 16, 1979, and in revised form on October 4, 1979.

Copyright © 1981 Rocky Mountain Mathematics Consortium