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SOME PROPERTIES OF RELATIVE PRINCIPAL COFIBRATIONS

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ABSTRACT. This paper studies the basic homotopy properties of relative principal cofibrations including a special suspension needed to extend the short exact sequences for relative mapping cones and attaching maps to a long exact sequence. Useful in classifying relative extensions.

Introduction. §1 contains the definition of a relative principal cofibration, an exact sequence of sets of homotopy classes and a group action useful in enumerating homotopy classes of extensions of a given map. This one sequence can be applied to all types of homotopy classes of extensions, from those relative to the domain of the map being extended to those relative to the base point. From this sequence and group actions, we can recover the result of Barcus-Barratt [1] as a special case. The results of §1 are dual to those of McClendon [5] and hence the proofs will be omitted. Complete details are given in Kruse [3].

A suspension operation $\bar{\Sigma}$ is defined in §2 and the homotopy equivalence between the relative mapping cone of $\bar{\Sigma} f$ and the suspension of the mapping cone of f is proven. Also, the exactness of the sequence

$$\cdots \leftarrow \bigoplus_i \pi_{r(i)+k}(Z) \xleftarrow{(\bar{\Sigma}^k f)^*} \pi_{n+k}(Z) \xleftarrow{\ i\ } [K(\bar{\Sigma}^k f), Z]^D$$

$$\leftarrow \bigoplus_i \pi_{r(i)+k+1}(Z) \xleftarrow{(\bar{\Sigma}^{k+1} f)^*} \pi_{n+k+1}(Z)$$

can be obtained as a special case of Corollary 2.10, where

$$f = \langle f_i \rangle : S^{r(1)} \vee S^{r(2)} \vee \cdots \vee S^{r(w)} \to D \vee S^n,$$

$\pi = \langle \mathrm{id}, * \rangle : D \vee S^n \to D$ and $\pi \circ f_i \sim 0$ for each i. $\bar{\Sigma}^k f = \langle \bar{\Sigma}^k f_i \rangle$ where $\bar{\Sigma}^k f_i$ is the k-fold one-sided suspension of f_i: $S^{r(i)} \to D \vee S^n$ as defined in (2.3). $K(\bar{\Sigma}^k f)$ is the ordinary mapping cone of $\bar{\Sigma}^k f$ and $f^*(\alpha)$ means $f^*\langle \check{z}, \alpha \rangle$ where $\check{z} : D \to Z$ is a fixed map. This sequence plays a central role in [4].

NOTATION. Top($C \to D$) will denote the category, whose objects are triples $(X, \check{x}, \hat{x})$ where $\check{x}: C \to X$, $\hat{x}: X \to D$ are continuous and $\hat{x}\check{x} = u$:

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