ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 11, Number 4, Fall 1981

α-CLOSURE IN FUZZY TOPOLOGY

ALBERT J. KLEIN

ABSTRACT. Let X be an L-fuzzy topological space, let $\alpha \in L$, and let A be a crisp subset of X. The α-closure of A is the set of points x for which $G(x) > \alpha$ implies $G(a) \neq 0$ for some $a \in A$ whenever G is fuzzy open. With appropriate restrictions on α (which always are satisfied if L is a chain), α-closure is a semi-closure operator but may not be a closure operator. Relations between α-closure and recently introduced α-level properties are studied and a characterization of α-closure in the fuzzy unit interval is obtained. The non-suitability of the fuzzy unit interval and fuzzy open unit interval follows as a simple corollary.

Introduction. Recently Gantner et al. [2] and Rodabaugh [4, 5] have studied L-fuzzy topological spaces by considering properties which a space may have to a certain degree or at a certain α-level, where α is a member of the underlying lattice. As part of this approach in [5], the concept of α-closure was introduced. It is the purpose of this paper to study α-closure in more detail as a closure operator, to examine its relations with other α-level properties, and to characterize it in Hutton's fuzzy unit interval [3].

Throughout this paper L will denote a completely distributive lattice with 0, 1 ($0 \neq 1$) and with an order-reversing involution $\alpha \to \alpha'$. As in [2], $L^c = \{\alpha \in L\colon \alpha$ is comparable to each $\beta \in L\}$ and $L^a = \{\alpha \in L^c\colon$ if $\beta > \alpha$ and $\gamma > \alpha$, then $\beta \wedge \gamma > \alpha\}$.

1. α-Closure as a semi-closure operator. Let (X, T) be an L-fuzzy topological space (L-*fts*). The following definition can easily be shown equivalent to the definition in [5].

DEFINITION 1.1. Let $\alpha \in L$-$\{1\}$ and let A be a crisp subset of X. $c_\alpha(A) = \{x\colon$ if $G \in T$ and $G(x) > \alpha$, then $G \wedge \chi_A \neq 0\}$.

Clearly $c_\alpha(\emptyset) = \emptyset$ and $A \subseteq c_\alpha(A)$ for every A. With a restriction on α one obtains the following lemma.

LEMMA 1.2. *Let $\alpha \in L^a - \{1\}$ and let $A, B \subseteq X$. Then $c_\alpha(A \cup B) = c_\alpha(A) \cup c_\alpha(B)$.*

AMS (MOS) (1970) Subject Classifications: Primary 54A05; Secondary 54D30.
Received by the editors on March 1, 1979, and in revised form on November 11, 1979.