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EQUICONVERGENCE OF INTERPOLATING PROCESSES

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1. **Introduction.** The following theorem is due to J. L. Walsh [3, p. 153].

THEOREM. *Let f be analytic for* $|z| < R$, $R > 1$. *Let* $p_n(z)$ *be the polynomial of degree n which coincides with f at the roots of the unity* $e^{2k\pi i/(n+1)}$, $k = 0, 1, \ldots, n$, *and let* $q_n(z)$ *be the n-th Taylor polynomial of f around the origin. Then* $p_n(z) - q_n(z) \to 0$, $n \to \infty$, *for* $|z| < R^2$.

It was a lecture of A. Sharma, in which he presented several extensions of this theorem, extensions obtained jointly by A. S. Cavaretta Jr., A. Sharma and R. S. Varga [1], that has directed our attention to this topic. In this paper we present a generalization of Walsh's theorem that goes in a different direction than the results in [1].

2. **Notation.** The infinite triangular matrix $[z_{kn}]$, $k = 1, 2, \ldots, n$; $n = 1, 2, \ldots$, where the entries z_{kn} are complex numbers, defines a Hermite interpolation process. We assume that we are given two such matrices, $[z_{kn}]$ and $[\tilde{z}_{kn}]$, and that $|z_{kn}| \leqq d$, $|\tilde{z}_{kn}| \leqq d$ for all k, n. We write

$$w_n(z) = \prod_{k=1}^{n}(z - z_{kn}) = \sum_{r=0}^{n} A_{n,r} z^{n-r}$$

and similarly we define the polynomials $\tilde{w}_n(z)$ and the coefficients $\tilde{A}_{n,r}$. If f is a function analytic in $|z| < R$, and $R > d$, then the interpolating polynomials to f based on the systems $[z_{kn}]$ and $[\tilde{z}_{kn}]$ are denoted by $p_n(z, f)$ and $\tilde{p}_n(z, f)$, respectively.

3. Let us formulate a heuristic principle. If two systems of interpolating nodes, $[z_{kn}]$ and $[\tilde{z}_{kn}]$, are "close", then the set of z's for which

$$p_n(z, f) - \tilde{p}_n(z, f) \to 0, \; n \to \infty, \tag{1}$$

is "large". In particular, it can be larger than the set on which f is analytic. This principle looks quite natural, because if the two systems are identical, then certainly (1) holds for all z.

We can look at Walsh's theorem in the light of this heuristic principle. Taylor's polynomials of f at the origin are the interpolating polynomials corresponding to the system of the nodes $[\tilde{z}_{kn}]$, where $\tilde{z}_{kn} = 0$ for every k

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