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BOUNDARY BEHAVIOR OF SPACES OF ANALYTIC FUNCTIONS

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0. **Introduction.** We de fine for $p \geqq 1$, $b > 0$, the space $M_{p,b}$ of function $f(z) = \sum_{n=0}^{\infty} \hat{f}(n) z^n$, analytic in the unit disc D, such that

$$\|f\|_{p,b} = \limsup_{r \to 1} (1 - r)^b \left[\int_0^{2\pi} |f(z)|^p d\theta / 2\pi \right]^{1/p} < \infty,$$
$$z = r \exp i\theta.$$

Two functions f and g are identified in $M_{p,b}$ whenever $\|f - g\|_{p,b} = 0$. We also define for $a > 0$, the space $M_{\infty,a}$ of functions $f(z)$, analytic in D such that

$$\|f\|_{\infty,a} = \limsup_{r \to 1} (1 - r)^a \max_{|z|=r} |f(z)| < \infty;$$

two functions f and g in $M_{\infty,a}$ are identified whenever $\|f - g\|_{\infty,a} = 0$, that is $f(r \exp i\theta) - g(r \exp i\theta) = o(1 - r)^a$, uniformly in θ.

For $b = 0$ a space $M_{p,b}$ reduces to a Hardy space H^p; for a description of the Hardy space see [**1**, **6**]. If f is in a Hardy space H^p, then $\|f\|_{p,b} = 0$ for all $b > 0$.

In addition to the obvious relations $M_{p,b} \subseteqq M_{q,b}$ for $p \geqq q$ we also have

$$M_{p,a-1/p} \subseteqq M_{q,a-1/q} \tag{1}$$

for $1 \leqq p \leqq q < \infty$, $a > 1/p$; moreover there exist constants C, C' such that

$$\begin{aligned} \|f\|_{\infty,a} &\leqq C\|f\|_{p,a-1/p} \\ \|f\|_{q,a-1/q} &\leqq C'\|f\|_{p,a-1/p} \end{aligned} \tag{2}$$

(cf. [**1**, p. 84]).

The relations (2) shows that if a function f is in a space $M_{p,a-1/p}$, $p \geqq 1$, $a > 1/p$, then $(1 - |z|)^a f(z)$ must remain bounded as z approaches a boundary point of D. In this note we will obtain restrictions on the values which $(1 - |z|)^a f(z)$ approaches as z approaches the boundary of D for functions f in a space $M_{p,a-1/p}$. We will also study topological properties of the $M_{p,a}$ spaces.

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