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TANGENT BUNDLE CONNECTIONS AND THE GEODESIC FLOW

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1. **Introduction.** If M is a Riemannian manifold there are several ways to induce a pseudo-Riemannian metric on the total space TM of the tangent bundle of M ([**10**], [**12**]). The present paper is concerned with the use of these natural structures to study the geodesic flow on the unit sphere bundle of M.

Our point of view is to study the dynamical properties of the geodesic flow in terms of certain spectral properties of the operator, "Lie differentiation in the direction of the geodesic vector field," defined in an appropriate space of sections. This operator decomposes into a sum of operators, one of which is the covariant derivative associated with the tangent bundle connection defined by an induced pseudo-Riemannian metric on TM, and the spectral properties follow from this decomposition.

In §2 and §3 we collect notation and previous results. §4 contains the decomposition of the Lie differentiation operator. The final sections, 5 and 6, contain the applications. In particular, we prove that the geodesic flow in the unit tangent bundle of a compact manifold of constant negative curvature is infinitesimally ergodic.

2. **The geodesic flow, spaces of sections and the adjoint representation.** Let M denote a smooth compact connected Riemannian manifold with metric tensor g. The geodesic flow G_t on the unit tangent bundle T_1M generates the geodesic vector field X which has local form

$$X(x, v) = (x, v, v, -\Gamma(v, v))$$

where $(x, v) \in T_1M$ and Γ is the vector valued bilinear form defined by the Levi-Civita connection.

If $T\pi$: $T^2M \to TM$ dedotes the derivative of π: $TM \to M$ we have the familiar commutative diagram of bundle maps:

$$\begin{array}{ccc} T^2M & \xrightarrow{T\pi} & TM \\ {\scriptstyle \pi_{TM}}\downarrow & & \downarrow{\scriptstyle \pi} \\ TM & \xrightarrow{\pi} & M \end{array}$$

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