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AMENABLE GROUPS FOR WHICH EVERY TOPOLOGICALLY LEFT INVARIANT MEAN IS RIGHT INVARIANT

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Let G be an amenable locally compact group. A. L. T. Paterson proved that

(∗) every topologically left invariant mean on $L^\infty(G)$ is (topologically) right invariant if and only if G has relatively compact conjugacy classes,

provided G is discrete or compactly generated. He also conjectured that (∗) holds for all amenable locally compact groups. We are unable to deal with this conjecture completely, but can verify it for σ-compact groups and, with the word in parentheses removed, for groups with equivalent left and right uniform structures. Thus we generalize both cases dealt with by Paterson. Our method of proof is quite different from Paterson's and relies heavily on results of W. R. Emerson and C. Chou.

1. **Preliminaries.** Let G be a locally compact group with the usual (complex-valued) function spaces $C(G)$ of continuous functions, $LUC(G) = LUC$ of functions $f \in C(G)$ for which

$$s \to L_s f\colon G \to C(G)$$

is (norm-) continuous and $L^\infty(G) = L^\infty$ of essentially bounded measurable functions. (Here, and later, $L_s f(t) = f(st) = R_t f(s)$.) These spaces are C^*-algebras. A continuous linear functional m on one of them is called a *mean* if

$$\|m\| = m(1) = 1$$

(where 1 sometimes stands for the function in LUC everywhere equal to 1). A mean m is called a *left (right) invariant mean*, shortened to LIM (RIM), if $m(L_s f) = m(f)(m(R_s f) = m(f))$ for all $s \in G$ and all f in the space under consideration; m is called a *topologically left (right) invariant mean*, shortened to TLIM (TRIM) if $m(p * f) = m(f)\,(m(f * p) = m(f))$ for all f in the space under consideration and all $p \in L^1(G)$ satisfying $\|p\|_1 = \int p(s)ds = 1$. (Here $\check{p}(s) = p(s^{-1})$ and ds denotes a (fixed) left

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