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ON THE INITIAL BOUNDARY-VALUE PROBLEM FOR NON-HOMOGENEOUS INCOMPRESSIBLE HEAT CONDUCTING FLUIDS

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ABSTRACT. The existence of a weak global solution of the following system of initial boundary-value problem:

$$\rho(u' + u\cdot\nabla u) - \nabla(\varepsilon(\rho)\nabla u) + \operatorname{grad} p = \rho f,$$
$$\operatorname{div}(u)=0 \quad \text{on } (0, T) \times G,$$
$$u(x, t) = 0 \quad \text{on } (0, T) \times \partial G,\ u(x, O) = u^0 \quad \text{on } G$$

and of

$$\rho' + u\cdot\operatorname{grad}\rho = 0,$$
$$\rho(x, t) > 0 \quad \text{on } (0, T) \times G,\ \rho(x, 0) = \rho^0 \quad \text{on } G$$

with

$$\rho(\theta' + u\cdot\operatorname{grad}\theta) - \nabla(\chi(\rho)\nabla\theta) = \rho g + h \quad \text{on } (0, T) \times G,$$
$$\theta(x, t) = 0 \quad \text{on } (0, T) \times \partial G,\ \theta(x, 0) = \theta^0 \quad \text{on } G$$

is established by the method of successive approximations.

The purpose of this paper is to show the existence of a weak global solution of the first initial boundary-value problem for non-homogeneous viscous, incompressible heat conducting fluids. Let u, ρ, θ be the velocity, the density and the temperature of the fluid respectively. The motion of the fluid is described by the initial boundary-value problem

$$\text{(0.1)} \quad \begin{aligned} &\rho\left(\frac{\partial u}{\partial t} + u\cdot\nabla u\right) - \nabla(\varepsilon(\rho)\nabla u) + \operatorname{grad} p = \rho f, \\ &\operatorname{div}(u) = 0 \quad \text{on } (0, T) \times G, \\ &u(x, t) = 0 \quad \text{on } (0, T) \times \partial G \text{ and } u(x, 0) = u^0(x) \quad \text{on } G, \end{aligned}$$

where G is a bounded open subset of $\mathbf{R}^3$.

The conservation of mass is expressed by

$$\text{(0.2)} \quad \begin{aligned} &\frac{\partial\rho}{\partial t} + u\cdot\operatorname{grad}\rho = 0,\ \rho(x, t) > 0 \quad \text{on } (0, T) \times G, \\ &\rho(x, 0) = \rho^0(x) \quad \text{on } G. \end{aligned}$$

The conservation of internal energy is described by the initial boundary value problem

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