ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 11, Number 1, winter 1981

WILDNESS AND FLATNESS OF CODIMENSION ONE SPHERES HAVING DOUBLE TANGENT BALLS

R.J. DAVERMAN* AND L.D. LOVELAND

Introduction. A round n-dimensional ball B_p is said to be *tangent* to an $(n - 1)$-sphere Σ in E^n at a point $p \in \Sigma$ if $p \in B_p$ and $\Sigma \cap B_p \subset \text{Bd } B_p$. If $\text{Int } B \subset \text{Ext } \Sigma$, B_p is called an *exterior tangent ball* and if $\text{Int } B_p \subset \text{Int } \Sigma$, B_p is an *interior tangent ball.* When Σ has both an interior and an exterior tangent ball at p, Σ is said to have a *double tangent ball* at p. If Σ has a certain class of tangent ball for each point of a subset K of Σ, then Σ is said to have this class of *tangent balls over K.* A *uniform* collection of round is one in which every ball has the same radius.

One suspects that the subject of double tangent balls first arose as a rigidly geometric potential analogue to smoothness; if an $(n - 1)$-sphere Σ has double tangent balls at each point, then it would seem to be embedded with a geometrically nice kind of curvature. This would form a basis for a conjecture that, in this context, the double tangent balls property implies flatness. In response to a question by Bing **[2]** concerning this conjecture in 3-space, Bothe **[3]** and Loveland **[17]** independently proved that a 2-sphere in E^3 is flat if it has double tangent balls at each of its points. Griffith **[15]** had earlier produced an affirmative answer to Bing's question provided the collection of double tangent balls was known to be uniform. The situation when $n = 3$ is best summarized by the following theorem, which, although not explicitly stated in **[17]**, follows from the proof there. This generalization is also apparent from Cannon's subsequently developed *-taming set theory (see Corollary 6 of **[8]**).

THEOREM A. *If Σ is a 2-sphere in E^3 that is locally flat modulo a closed subset W of Σ and if Σ has double tangent balls over W, then Σ is flat.*

The examples from §1 show the impossibility of such a theorem for a codimension one sphere in E^n with $n > 3$; in fact, Theorem A does not generalize to $n > 3$ even with the added hypothesis that Σ has uniform double tangent balls over W. These examples stand as circumstantial evidence of the still unauthenticated possibility that an $(n - 1)$-sphere in E^n $(n < 3)$ with double tangent balls everywhere may fail to be flat.

Nevertheless there are interesting facts about higher dimensional

*Research supported in part by NSF Grant MC576–07274.

Received by the editors on January 16, 1979.