ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 10, Number 4, Fall 1980

ORDER CONVERGENCE IN LATTICES

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0. **Introduction.** There are two essentially different possibilities to define convergence of a real sequence: the topological one, using open neighbourhoods, and the order-theoretical one, involving the notions of limit inferior and limit superior. The latter can be generalized to sequences or—since sequences are often inadequate—to nets or filters in arbitrary partially ordered sets. The net-theoretical generalization was developed by G. Birkhoff [**1**], O. Frink [**3**], B.C. Rennie [**11**], and others. Some years later, about 1954, the study of order convergence in terms of filters was started by A.J. Ward [**13**]. Ward's method was continued and generalized by D.C. Kent (cf. [**7**, **8**, **9**]) whose general filter-convergence theory provides an elegant and powerful method to describe and develop the theory of order convergence. However, until now, nearly all deeper results have been formulated and derived in the language of nets, often requiring rather complicated proof techniques (see, for example, [**1**], [**5**, **6**], [**3**], [**11**]). Our purpose is to unify (and in some cases to correct) various results from the literature, and to complete them by giving several generalizations and new results, all formulated in the language of filters.

In §1, we compose the most important definitions and abbreviations concerning partially ordered sets and convergence theory.

In §2, we give several alternative characterizations of order convergence and show that it is always a localized convergence relation. Since it is well known that, in general, order convergence fails to be a limitierung, the question arises under which circumstances it is a limitierung, a pretopological or a topological convergence relation. For lattices, we answer this in §4.

A useful help for these investigations are the so-called conoids (cf. [**6**]) and ray filters (being in one-to-one correspondence to conoids): to any filter $\mathfrak{F}$, we can assign a ray filter such that $\mathfrak{F}$ order-converges to a point x if and only if the corresponding ray filter does. Hence, order convergence is completely described by the behaviour of ray filters (or conoids, respectively). A special class of ray filters is formed by the so-called interval filters, which possess a base of (closed bounded) intervals: the interval filters are just the bounded ray filters. Since every order-

Received by the editors on July 6, 1978.