ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 10, Number 4, Fall 1980

OSCILLATION THEORY FOR GENERALIZED SECOND-ORDER DIFFERENTIAL EQUATIONS

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1. **Introduction.** We will consider the generalized vector-matrix differential system

$$
\begin{aligned}
&R(t)u'(t) = v(t) \\
(1) \qquad &v(t) = v(a) - \int_a^t dM(t)u(t),\ t \in [a, \infty),
\end{aligned}
$$

where the n-dimensional vector-valued function $u(t)$ is assumed to be absolutely continuous on compact subintervals of $[a, \infty)$, and the real $n \times n$ matrices R and M satisfy

$$
\begin{aligned}
(2) \qquad &R^* = R,\ M^* = M,\ R(t) > 0 \text{ (positive definite) for all } t \in [a, \infty), \\
&R \text{ and } R^{-1} \text{ are locally } L^\infty, \text{ and } M \text{ is locally of bounded variation.}
\end{aligned}
$$

By A^* we mean the transpose of the matrix A. The matrix-valued Riemann-Stieltjes integral of (1) is a direct generalization of the scalar Riemann-Stieltjes integral. The associated properties are direct consequences of the properties of the scalar integral (cf. Reid **[14]**).

If there is a function $Q(t)$ that is integrable on compact subintervals of $[a, \infty)$ such that

$$M(t) = M(t_0) + \int_{t_0}^t Q(s)\, ds$$

for some $t_0 \in [a, \infty)$, then (1) reduces to the vector-matrix differential equation

$$(3) \qquad (Ru')' + Qu = 0.$$

If we define for $n = -1, 0, 1, 2, \ldots$

*The first author received support for this research under N. S. F. Grant MCS 77–28268.

AMS (MOS) *subject classifications* (1970); Primary 34C10; Secondary 34B25.
Received by the editors on October 20, 1978.