ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 10, Number 4, Fall 1980

BANACH SPACES WHICH ARE NEARLY UNIFORMLY CONVEX

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ABSTRACT. A property which generalizes uniform convexity is defined in terms of sequences. Its relationships to uniform convexity and to weak and norm convergence on spheres are investigated.

1. **Introduction.** Let X be a (real) banach space with norm $\|\cdot\|$, let $B_\delta(x)$ (respectively, $\bar{B}_\delta(x)$) denote the open (closed) ball with center x and radius δ, and let $\mathrm{co}(A)$ ($\overline{\mathrm{co}}(A)$) denote the convex hull (closed convex hull) of a set A.

We will say that the norm is a *Kadec-Klee* (KK–)*norm* provided on the unit sphere sequences converge in norm whenever they converge weakly. (This is property (H) in [2].) An equivalent formulation is the following.

$$\text{(KK)}: \left.\begin{array}{l} (x_n)_{n=1}^\infty \subset \bar{B}_1(0) \\ x_n \to x \text{ wkly} \\ (x_n)_{n=1}^\infty \textit{ not } \text{norm Cauchy} \end{array}\right\} \Rightarrow \|x\| < 1.$$

For notation, given a sequence (x_n) we let

$$\mathrm{sep}(x_n) = \inf\{\|x_n - x_m\| : m \neq n\}.$$

If (x_n) is not norm-Cauchy, then for some subsequence (y_n) we must have $\mathrm{sep}(y_n) > 0$. The above definition can be reformulated as follows.

$$\text{(KK)}: \left.\begin{array}{r} (x_n) \subset \bar{B}_1(0) \\ x_n \to x \text{ wkly} \\ \mathrm{sep}(x_n) > 0 \end{array}\right\} \Rightarrow \|x\| < 1.$$

This formulation suggests the following two successively stronger notions.

The norm will be called *uniformly Kadec-Klee* (UKK) if for every $\varepsilon > 0$ there exists $\delta < 1$ such that

$$\text{(UKK)}: \left.\begin{array}{l} (x_n) \subset \bar{B}_1(0) \\ x_n \to x \text{ wkly} \\ \mathrm{sep}(x_n) \geqq \varepsilon \end{array}\right\} \Rightarrow x \in B_\delta(0).$$

Presented to the American Mathematical Society January, 1979, Biloxi, Mississippi.
Received by the editors on April 2, 1979.