ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 10, Number 4, Fall 1980

COUNTABILITY PROPERTIES OF FUNCTION SPACES

R. A. MCCOY

ABSTRACT. A study is made of those function spaces which have such properties as first and second countability, separability, the Lindelöf property, and the properties of $\aleph_0$-spaces and cosmic spaces.

Key words and phrases: Function spaces, first countable, metrizable, separable, Lindelöf, $\aleph_0$-space, cosmic space.

The purpose of this paper is to organize and extend results concerning countability properties of function spaces. The term "countability properties" refers to those topological properties which involve some countable set in their definitions, such as first and second countable spaces, separable spaces, or Lindelöf spaces.

We shall be concerned with function spaces having topologies which are of "closed-open" form. That is, if X and Y are topological spaces, and if $C(X, Y)$ denotes the space of continuous functions from X into Y, then a topology of "closed-open" form is one generated by the sets of the form $[C, V] \equiv \{f \in C(X, Y) \mid f(C) \subseteq V\}$, where C is from some predetermined collection of nonempty closed subsets of X, and V is open in Y. We will call Γ a *closed collection from* X (*compact collection from* X, respectively) if it is a family of nonempty closed (compact, respectively) subsets of X; and we will use the notation $C_\Gamma(X, Y)$ to denote the space $C(X, Y)$ with the topology generated by the subbase $\{[C, V] \mid C \in \Gamma \text{ and } V \text{ is open in } Y\}$. When Γ consists of the singleton subsets of X, then the topology on $C_\Gamma(X, Y)$ is the *topology of pointwise convergence* and will be specifically denoted by $C_\pi(X, Y)$. The symbol π will then be used to mean the family of all singleton subsets of X. Also when Γ consists of the nonempty compact subsets of X, then the topology on $C_\Gamma(X, Y)$ is the *compact-open topology* and denoted by $C_\kappa(X, Y)$, and κ will be used to mean the family of all nonempty compact subsets of X.

The range space Y can naturally be embedded in $C_\Gamma(X, Y)$ by associating points of Y with the constant functions; and if Y is a Hausdorff space, then this is a closed embedding. Therefore, for a hereditary property (or a

AMS-MOS *subject classification*: Primary 54C35; secondary 54D99.
Received by the editors on January 24, 1977 and in revised form on February 6, 1979.