

CHARACTERIZATIONS OF SOME GENERALIZED COUNTABLY COMPACT SPACES AS IMAGES OF M -SPACES

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ABSTRACT. The main theorems of this paper characterize regular $w\mathcal{A}$ -spaces, wM -spaces, and wN -spaces as almost-open images of regular M -spaces. For example, it is proved that a regular space Y is a wN -space if and only if there exist a regular M -space X , with $w\mathcal{A}$ -function g , and an almost-open mapping f from X onto Y which is a wP -mapping relative to g . Similar theorems are proved for wM -spaces and for $w\mathcal{A}$ -spaces. Moreover, for the wN -space case, it is shown that if f is an almost-open mapping from any space X onto another space Y , then for Y to be a wN -space it is necessary that f be a wP -mapping relative to some function g .

1. Introduction. Characterizing spaces as images of other spaces with more structure is an old and much investigated problem in topology. For example, there are many interesting theorems characterizing certain spaces as images of metric spaces. One can pursue extensions and analogues to these results in various directions. One such direction is to vary the domain from metric to a more general class of spaces, for example M -spaces. The concept of an M -space has emerged as an important generalization of metric spaces, and in fact has been investigated within this framework. Several classes of spaces have been characterized as images of M -spaces, particularly see Chiba [3] and Nagata [15] and [16].

In this paper we continue this study by characterizing $w\mathcal{A}$ -spaces, wM -spaces, and wN -spaces as almost-open continuous images of M -spaces. These classes of spaces have been studied by various mathematicians recently. For example, $w\mathcal{A}$ -spaces are studied in [1], [2], [4], [6], [7], and [8]; wM -spaces in [7], [10], and [11]; and wN -spaces are introduced in [7]. These spaces have useful applications in metrization theory. The class of M -spaces was introduced in [12] and is studied in [9], [13], [14], [15], [16], and [17].

The present work was motivated by Chiba's [3] characterization of a q -space as an almost-open continuous image of a regular M -space. Indeed, we make use of his basic construction. Also, to some extent, our defini-

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