ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 10, Number 4, Fall 1980

FUNCTORIAL TOPOLOGIES ON ABELIAN GROUPS

D. BOYER AND A. MADER

1. **Introduction.** B. Charles [3] introduced the concept of "functorial topology" which includes the ubiquitous p-adic and Z-adic topologies. He proposed a method for constructing such topologies which was slightly generalized by Fuchs [4], Vol I, p. 33. Closer inspection shows that this method amounts to specifying the class of discrete groups and furnishing all other groups with the coarsest topology required to make all homomorphisms continuous. The "discrete classes" satisfy certain closure properties and studying these classes enables us to partially solve Fuchs' Problem 2, [4], asking for a description of functorial linear topologies. There is no bijective correspondence between functorial topologies and discrete classes — examples are readily available (see 2.8) — but there is a bijective correspondence between the "minimal functorial topologies" obtained via the Charles-Fuchs construction and discrete classes (Theorem 2.5). We call a functorial topology "ideal" if in addition to having continuous homomorphisms all epimorphisms are open maps. This is true for the p-adic and Z-adic topologies, for example. We obtain a bijective correspondence between linear ideal functorial topologies and "ideal" discrete classes. The latter are satisfactorily characterized (3.5, 3.11, 3.19) except for one nasty case.

In describing the ideal discrete classes we adopt the methods of Balcerzyk's description [1] of "classes" (= Serre classes). A Serre class is a class of groups closed under subgroups, homomorphic images, and extensions; while an ideal discrete class is only closed under subgroups, homomorphic images, and finite direct sums. Because of this difference all of Balcerzyk's results had to be modified, and we present our own proofs and arrangement. The later theorems on mixed groups have no analogues in [1].

In §2 we discuss functorial topologies and discrete classes. In §3 we outline without proofs the steps involved in describing the ideal discrete classes, and in §4 we supply the proofs. In a final section we discuss possible further work and related literature.

We use the standard notation of Fuchs [4], and all unexplained terminology and symbols can be found there. We write maps on the right. P is the set of all primes; p always denotes a prime; N is the set of positive

Received by the editors on April 14, 1978, and in revised form on December 14, 1978.