ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 10, Number 4, Fall 1980

GLOBAL PROPERTIES OF SPACES OF *AR*'s

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ABSTRACT. We study the hyperspace (denoted AR_h^X) of compact absolute retract subsets of certain finite-dimensional compacta X. The topology of AR_h^X is induced by Borsuk's homotopy metric. We show AR_h^X is contractible if X is pseudoisotopically contractible. We show AR_h^X is simply-connected if X is a sphere of dimension greater than 1.

1. **Introduction.** Let X be a finite-dimensional compactum and let 2_h^X be the space of nonempty compact ANR subsets of X introduced by Borsuk [2]. If d is a metric for X, the topology of 2_h^X is induced by the *homotopy metric* d_h, which may be described as follows: $d_h(A_i, A) \to 0$ if and only if

a) $d_s(A_i, A) \to 0$, where d_s is the well-known Hausdorff metric, and

b) for every $\varepsilon > 0$ there is a $\delta > 0$ such that every A_i-subset of diameter less than δ contracts to a point in an A_i-subset of diameter less than ε.

We let AR_h^X be the subspace of 2_h^X consisting of the members of 2_h^X that are absolute retracts (AR's). Since AR_h^X is open and closed in 2_h^X ([2], p. 200), AR_h^X is a union of components of 2_h^X.

Let I denote the interval [0, 1]. We will use the following lemmas.

LEMMA 1.1. ([1], 4.2, p. 43). *If $A \in 2_h^X$ and f: $A \times I \to X$ is an isotopy, then the function $g: I \to 2_h^X$ defined by $g(t) = f_t(A)$ is continuous.*

LEMMA 1.2. ([4], 2.1). *Let U be open in X. Then $\{A \in 2_h^X | A \subset U\}$ is open in 2_h^X.*

2. We will denote by $s(A, \delta, \varepsilon)$ the words "every A-subset of diameter less than δ contracts to a point in an A-subset of diameter less than ε." We prove the following lemma.

LEMMA 2.1. *Let X and Y be finite-dimensional compacta with $X \subset Y$. Let f: $X \times I \to Y$ be an isotopy. Then the induced function f_*: $2_h^X \times I \to 2_h^Y$ defined by $f_*(A, t) = f_t(A)$ is continuous.*

AMS (MOS) 1970 subject classification: Primary 54B20, 54F40; Secondary 57A05, 57A10, 57A15.

Key words and phrases: Finite-dimensional compactum, Hausdorff metric, homotopy metric.

Received by the editors on July 26, 1977.