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SOME ELEMENTARY PROOFS OF BASIC THEOREMS IN THE COHOMOLOGY OF QUASI-COHERENT SHEAVES

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In his fundamental paper [5], Serre initiated the study of the Čech cohomology of coherent sheaves on (separated) algebraic varieties with their Zariski topology. He proved that coherent sheaves on affine varieties have zero higher cohomology groups. Using this theorem, he showed that one may compute the cohomology of coherent sheaves by using the more down-to-earth Čech cohomology of any affine open covering of the algebraic variety.

Grothendieck [2] defined a cohomology theory of sheaves on any topological space by injective resolutions. In this theory, the emphasis is on the long exact sequence of cohomology arising from any short exact sequence of sheaves. A modification of this theory was presented by Godement [1] making use of canonical flabby resolutions.

In algebraic geometry, Grothendieck [3] greatly extended Serre's results to quasi-coherent sheaves on schemes. His argument appears to be a direct translation of Serre's to the more general context and freely employs spectral sequences.

In the first section of this paper, I will give a proof of Serre's affine vanishing theorem. My proof of this theorem makes no use of the Čech cohomology and deals with the general cohomology theory. The main step is a direct proof of the effaceability of the cohomology of a quasi-coherent sheaf on an affine scheme. I prefer to reserve Čech cohomology for its proper place as one of many possible ways of computing the sheaf cohomology.

In section two, I develop formalities about direct limits of sheaves on quasi-noetherian topological spaces. These are topological spaces with strong finiteness properties. Such spaces arise in algebraic geometry as the underlying spaces of quasi-compact, quasi-separated schemes; e.g. affine schemes. These simple formalities are used quite effectively to deduce the results of section three.

In section four, I show that the sheaf cohomology coincides with Čech cohomology in a reasonable situation. The same type of argument yields easily a special case of the Künneth formula.

After many propositions, I will refer to the closest approximation that

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