

SEMI-STABLE KERNELS OF VALUATED GROUPS

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ABSTRACT. A characterization of semi-stable kernels of valuated abelian groups is given.

1. **Introduction.** The concept of valuated groups has recently been developed extensively by Richman and Walker [2]. (Throughout this paper, the term "group" will mean abelian group.) If A is a subgroup of the group B , the p -height function of B restricted to A gives rise to a valuation on A . This relation has been quite useful in determining the structure of certain classes of groups. (For a more detailed discussion, see the introduction of [2].) Richman and Walker [1] have developed a theory of Ext in pre-abelian categories, and have applied this in [2] to valuated groups. The notions of semi-stable kernels and semi-stable cokernels are fundamental to this theory. While semi-stable cokernels are classified in a satisfactory way in [2], the question of classifying semi-stable kernels is left open. In this paper, a characterization of semi-stable kernels is given.

2. **Valuated Groups.** In this section, we summarize some definitions and results on valuated groups. Most of this discussion originated in [2]. Let G be an abelian group and p be a prime. The p -height function on G is characterized by

$$h_p x = \sup\{h_p y + 1 : x = py\}$$

where $h_p x$ is either an ordinal or ∞ . We say $\infty < \infty$ and $\alpha < \infty$ for any ordinal α .

DEFINITION. Let A be a group and p be a prime. A p -valuation v_p on A is a function on A satisfying the following properties:

- 1) $v_p x$ is an ordinal or ∞
- 2) $v_p(x + y) \geq \min(v_p x, v_p y)$
- 3) $v_p px > v_p x$
- 4) $v_p nx = v_p x$ if n is not divisible by p .

If A is a subgroup of B , then the p -height function on B , restricted to A , is a p -valuation on A . We will restrict our study to p -local valuated groups,