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FIELDS WITH FREE MULTIPLICATIVE GROUPS MODULO TORSION

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Fields whose multiplicative groups are free modulo roots of unity occur in several familar settings. Not only are finite algebraic number fields of this type, but so is every field that is finitely generated over its prime subfield. In addition, there are interesting examples which are infinitely generated. A theorem of Schenkman [6] shows that if K is generated over the rational numbers by a family of algebraic elements of bounded degree, then the multiplicative group K^* is free modulo torsion. Likewise, it follows from [5] that the same conclusion holds if K is the maximal cyclotomic extension of a finite algebraic number field.

The main purpose of this paper is to generalize two theorems in [5] from countable fields to arbitrary fields. These theorems were based on the last two examples mentioned above, and were shown by utilizing Pontryagin's theorem on countable torsion-free abelian groups. We can now avoid this utilization by a more thorough consideration of the field structures involved. We shall prove the following generalization of the theorem of Schenkman.

THEOREM 1. *Assume that F is a field such that for every finite extension field E, E^* is free modulo torsion. Let K be any field generated over F by algebraic elements whose degrees over F are bounded. Then K^* is free modulo torsion.*

In the second result, K is taken to be an (infinite) Galois extension of F with abelian Galois group.

THEOREM 2. *Assume that F is a field such that for every finite extension field E, E^* is free modulo torsion and E contains only finitely many roots of unity. Let K be any abelian extension field of F. Then K^* is free modulo torsion.*

We point out that it is not enough in either theorem to assume that F^* alone is free modulo torsion (see Example 1 in [5]). Furthermore, it is not enough in Theorem 2 to assume that F alone contains only finitely many roots of unity. To see this, let C be the maximal cyclotomic extension of the rational numbers, let F be the maximal real subfield of C, and let K

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