ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 10, Number 3, Summer 1980

C^*-ALGEBRAS OF FUNCTIONS ON DIRECT PRODUCTS OF SEMIGROUPS

H. D. JUNGHENN

ABSTRACT. Berglund and Milnes, generalizing results of deLeeuw and Glicksberg, have shown that if S_1 and S_2 are semitopological semigroups with right and left identities respectively, then the almost periodic (AP) compactification of the direct product $S_1 \times S_2$ is the direct product of the AP compactifications of S_1 and S_2—in symbols $(S_1 \times S_2)^{AP} = S_1^{AP} \times S_2^{AP}$. They also showed that the analog of this result holds in the weakly almost periodic (WAP) case if S_1 is a compact topological group. In this paper we extend these results, first by replacing the spaces AP and WAP by more general C^*-algebras of functions, and second by considering direct products of arbitrarily many semigroups. Several general theorems are proved from which the following corollaries may be derived: If S_1 is a dense subsemigroup of a compact topological group G then $(S_1 \times S_2)^{WAP} = G \times S_2^{WAP}$ and $(S_1 \times S_2)^{LUC} = G \times S_2^{LUC}$. If $\{S_i : i \in I\}$ is a family of semitopological semigroups with identities and $S = \Pi\{S_i : i \in I\}$, then $S^{AP} = \Pi\{S_i^{AP} : i \in I\}$ and $S^{SAP} = \Pi\{S_i^{SAP} : i \in I\}$.

1. **Introduction.** Let S_1 and S_2 be semitopological semigroups, $S = S_1 \times S_2$ their direct product, and F a sub-C^*-algebra of $C(S)$. Define

$$(1) \qquad F_1 = \{f(\cdot, s_2) : f \in F, s_2 \in S_2\}, F_2 = \{f(s_1, \cdot) : f \in F, s_1 \in S_1\}.$$

Suppose that S has a right topological F-compactification S^F (as defined below) and that S_i has a right topological F_i-compactification $S_i^{F_i}$ $(i = 1, 2)$. We wish to determine conditions under which S^F is (canonically isomorphic to) the direct product of $S_1^{F_1}$ and $S_2^{F_2}$—in symbols,

$$(2) \qquad S^F = S_1^{F_1} \times S_2^{F_2}.$$

Special cases of (2) have been verified by several authors. In **[6]** it was shown that (2) holds for the case $F = AP(S)$ (hence $F_i = AP(S_i)$) when S_i is a commutative topological semigroup with identity $(i = 1, 2)$. The restrictions of commutativity and joint continuity of multiplication were later removed in **[8]** using the device of tensor products. In **[4]** it was shown that S_1 and S_2 need only have right and left identities respectively.

The situation is less well-behaved in the weakly almost periodic case. For example, if $S_1 = S_2$ is any commutative topological semigroup with

Received by the editors on March 1, 1978.