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ISOMETRY AND FIXED POINT THEOREMS FOR ASYMPTOTICALLY EXPANSIVE MAPPINGS IN COMPACT SPACES

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ABSTRACT. An isometry theorem in a compact metric space that generalizes a result of Freudenthal and Hurewicz is proved. A common fixed point theorem for a family of asymptotically expansive mappings of a compact set in a Banach space is then deduced.

1. **Introduction.** Let M be a metric space. A mapping $T: M \to M$ is called nonexpansive if $d(Tx, Ty) \leqq d(x, y)$ for all $x, y \in M$. In 1963, DeMarr proved the following theorem.

THEOREM A. *Let B be a Banach space and let X be a nonempty compact convex subset of B. If F is a nonempty commutative family of nonexpansive mappings of X into itself, then the family F has a common fixed point in X.*

A mapping T is said to be asymptotically nonexpansive if for each $x, y \in M$,

$$d(T^i x, T^i y) \leqq k_i d(x, y), \qquad i = 1, 2, 3, \dots,$$

where k_i is a fixed sequence of positive numbers such that $k_i \to 1$ as $i \to \infty$. The concept of "asymptotic nonexpansiveness" was introduced by Goebel and Kirk in [5]. In 1974, Goebel, Kirk and Thele [6] showed that the conclusion of Theorem A is still valid if F is merely assumed to be a commutative family of asymptotically nonexpansive mappings. They also proved (see [6]) that an asymptotically nonexpansive mapping of a compact metric space onto itself must be an isometry. Since the inverse of a nonexpansive (asymptotically nonexpansive) mapping is expansive (asymptotically expansive), one is naturally led to the study of expansive and asymptotically expansive mappings whose precise definitions are given below.

DEFINITION. Let M be a metric space. A mapping $T: M \to M$ is said to be expansive if $d(Tx, Ty) \geqq d(x, y)$ for all $x, y \in M$. And it is said to be asymptotically expansive if for each $x, y \in M$,

$$d(T^i x, T^i y) \geqq k_i d(x, y) \qquad i = 1, 2, 3, \dots,$$

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