ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 10, Number 3, Summer 1980

STRONGLY EXPOSED POINTS IN $L^p(\mu, E)$

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ABSTRACT. A sufficient condition is given for a function to be a strongly exposed point of the unit ball of $L^p(\mu, E)$ for any Banach space E, $1 < p < \infty$. It is then shown that the unit ball of $L^p(\mu, E)$ is the closed convex hull of the "simple strongly exposed points" if E has the Radon-Nikodym property.

Sundaresan **[3]** (see also Turett and Uhl **[6]**) showed that if E is a Banach space with the Radon-Nikodym property (RNP) then the space $L^p(\Omega, \Sigma, \mu, E) \equiv L^p(\mu, E)$ $(1 < p < \infty)$ also has RNP. One corollary of this result is that the unit ball of $L^p(\mu, E)$ is the closed convex hull of its strongly exposed points. For this reason it was suggested by J. J. Uhl that it would be useful to have available a characterization of these functions.

In **[1, 4** and **5]** the problem of characterizing the extreme points of the unit ball of $L^p(\mu, E)$ was considered and, with modest restrictions on E and (Ω, Σ, μ), it was shown that f is an extreme point if and only if $\|f\|_p = 1$ and for almost all $t \in \{t \mid f(t) \neq 0\}$, $f(t)/\|f(t)\|$ is an extreme point of the unit ball of E. This suggests a similar characterization for strongly exposed points; Theorem 1 gives a sufficient condition for f to be strongly exposed. We were unable to obtain the necessity, but got something a little better in a way (Theorem 2); namely, that the unit ball of $L^p(\mu, E)$ is the closed convex hull of the "simple strongly exposed points" if E has RNP. We assume throughout that (Ω, Σ, μ) is a finite measure space, U denotes the unit ball of E and E^* the dual of E. If $f: \Omega \to E$, $|f|(t) = \|f(t)\|$.

A point $x \in U$ is said to be strongly exposed by $x^* \in E^*$ if $x^*(x) = \|x^*\| = 1$, and any sequence $\{x_n\} \subset U$ for which $x^*(x_n) \to 1$ satisfies $\|x_n - x\| \to 0$. We state the following simple modification of the definition for later reference and omit its easy proof:

LEMMA 1. *Let $x \in E$ and $x^* \in E^*$ be such that $x^*(x) = \|x\| = \|x^*\| = 1$. Suppose every sequence $\{x_n\} \subset U$ with $x^*(x_n - x) \to 0$ has a subsequence converging to x. Then x^* strongly exposes x.*

For any unfamiliar notation or terminology we refer the reader to **[0]**.

THEOREM 1. *Let $f \in L^p(\mu, E)$, $1 < p < \infty$, and $\|f\|_p = 1$. Put $S =$*

Received by the editors on February 7, 1978.