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SOME TOTALLY REAL SUBMANIFOLDS IN A QUATERNION PROJECTIVE SPACE

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0. **Introduction.** Let HP^m be the (real) $4m$-dimensional quaternion projective space. On totally real submanifolds in HP^m, [1] has established some fundamental concepts and formulas. In this paper we employ some techniques developed in [2] and [4] and prove the following theorem.

THEOREM. *Let HP^m be the (real) $4m$-dimensional quaternion projective space of constant quaternion sectional curvature $c > 0$. Let N be an n-dimensional compact totally real minimal submanifold of HP^m. If the sectional curvature γ of N satisfies $\gamma \geqq (n-1)c/4(2n-1)$, then either N is totally geodesic in HP^m or $n = 2$, $m \geqq 4$ and N is the Veronese surface in HP^m with positive constant curvature $c/12$.*

1. **Preliminaries.** Let HP^m be a quaternion projective space with real dimension 4m. On HP^m there exists a 3-dimensional vector space V of tensors of type (1.1) with local basis of almost Hermitian structure I, J, K such that

$$\text{(a)} \quad IJ = -JI = K,\ JK = -KJ = I,\ KI = -IK = J,$$
$$I^2 = J^2 = K^2 = -1;$$
$$\text{(b)} \quad \tilde{\nabla}_x I = r(x)J - q(x)K,\ \tilde{\nabla}_x J = -r(x)I + p(x)K,$$
$$\tilde{\nabla}_x K = q(x)I - p(x)J$$

for some functions $p(x)$, $q(x)$, $r(x)$ on HP^m, where $\tilde{\nabla}$ is the connection on HP^m.

Let X be a unit vector on HP^m. Then X, IX, JX and KX form an orthonormal frame. Let $Q(X)$ be the 4 plane spanned by them. For X, Y on HP^m, if $Q(X)$ and $Q(Y)$ are orthogonal, the plane $\pi(X, Y)$ spanned by X and Y is called a *totally real plane*. Any 2-plane in some $Q(X)$ is called a *quaternion plane*. The sectional curvature of a quaternion plane π is called the *quaternion sectional curvature* of π. The quaternion sectional curvature of HP^m is a constant $c > 0$. HP^m is thus called a *quaternion-space-form*.

Let g be the Riemann metric on HP^m. Then the curvature tensor $\tilde{R}$ of HP^m is given by [3].

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