ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 10, Number 3, Summer 1980

CONVOLUTIONS AND GROWTH NUMBERS OF ANALYTIC FUNCTIONS

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ABSTRACT. The convolution of two analytic functions $f(z) = \sum_0^\infty a_n z^n$, $|z| < R$, $g(z) = \sum_0^\infty b_n z^n$, $|z| < S$ is defined as $(f*g)(z) = \sum_0^\infty a_n b_n z^n$, $|z| < R^*$. The aim of the paper is to establish a relationship between the growth numbers of f, g and $f*g$. The growth number $\rho(f)$ of an analytic function is defined as $\limsup (\log^+ \log^+ M(r, f)/\log (R/(R-r))$, as $r \to R^-$, where $M(r, f)$ is the maximum modulus function associated with f.

1. **Introduction.** Throughout the paper $A(R)$ will denote the class of functions $f = f(z) = \sum_{n=0}^\infty a_n z^n$ analytic in the disc $|z| < R$, where $R^{-1} = \lim \sup_{n\to\infty} |a_n|^{1/n}$. Also, we shall assume that $0 < R < \infty$ and $\sup_n(|a_n| R^n) = \infty$.

Let $g \in A(S)$ and $g = g(z) = \sum_{n=0}^\infty b_n z^n$. The convolution or Hadamard product of f and g is defined by the power series $(f*g)(z) = \sum_{n=0}^\infty a_n b^n z^n$. This new function is clearly analytic on the disc $|z| < R^*$ for some $R^* \geqq 0$. It is easy to show that $R^* \geqq RS$ and the product is commutative.

The measure of growth for any $f \in A(R)$ is indicated by the real number $\rho(f)$, $0 \leqq \rho(f) \leqq \infty$, which is determined as follows.

$$(*) \qquad \rho(f) = \lim_{r\to R} \sup (\log^+ \log^+ M(r, f)/\log (R/(R-r)),$$

where $\log^+ x = \max(\log x, 0)$ and $M(r, f) = \max_\phi |f(re^{i\phi})|$, the maximum modulus of f. The real number $\rho(f)$, in analogy to entire functions, may be called the "order of f". But we prefer to call it the "growth number of f". Likewise, we define the lower growth number $\lambda(f)$ of f by (*) with limit inferior in place of limit superior.

The object of this note is to relate the growth number of a convolution with the growth numbers of its component functions. To achieve this goal we will establish some allied results which are interesting and are used in proofs of main results.

We shall use the following definitions and notations. We define the functions:

Received by the editors on February 7, 1978, and in revised form on November 6, 1978.

AMS(MOS) subject classifications (1970). Primary 30A10, 30A64.

Key words and phrases: Hadamard product, Convolutions, Growth, orders.