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ČECH THEORY: ITS PAST, PRESENT, AND FUTURE

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ABSTRACT. We survey the development of Čech Theory with an eye towards certain recent developments in the homotopy theory of pro-spaces.

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1. **Introduction.** The main problem in topology is the classification of topological spaces up to homeomorphism. Recall that two topological spaces X and Y are said to be homeomorphic if there exist continuous maps $f: X \to Y$ and $g: Y \to X$ such that $f \circ g = 1_Y$ and $g \circ f = 1_X$. This problem is very difficult. As is usual in mathematics, when one is faced by a very difficult problem, one first tries to solve an easier problem. Hence arises homotopy theory. The main problem in homotopy theory is the classification of topological spaces up to homotopy equivalence. Recall that two continuous maps $f, g: X \to Y$ are said to be homotopic, denoted $f \simeq g$, if there exists a continuous map $F: X \times [0, 1] \to Y$ such that $F(x, 0) = f(x)$ and $F(x, 1) = g(x)$. Two topological spaces X and Y are now said to be homotopy equivalent (or to have the same homotopy type) if there exists continuous maps $f: X \to Y$ and $g: Y \to X$ such that $f \circ g \simeq 1_Y$ and $g \circ f \simeq 1_X$. This is still too hard a problem in general. So one first attacks the problem for a particularly nice class of topological spaces, namely compact polyhedra. Poincaré was the first person to systematically study the classification problem for polyhedra. In [136], Poincaré associated certain numerical invariants (namely, the Betti and torsion numbers) to a complex. These invariants are homotopy type invariants and can often be used to show that two complexes do not have the same homotopy type. During the late 1920's, under the influence of E. Noether, Poincare's constructions were reinterpreted as defining the integral homol-

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