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ROTATING CHAIN FIXED AT TWO POINTS VERTICALLY ABOVE EACH OTHER

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0. **Introduction.** In 1955 Kolodner [1] treated the problem of a chain fixed at one point, the other end being free. Using methods characteristic of ordinary differential equations, he showed that there are infinitely many branches of solutions with one, two, three, $\cdots$ nodes bifurcating from the zero solution, at angular velocities ω_i, the latter sequence tending to infinity. By the zero or trivial solution we mean the solution for which the chain rotates at any angular velocity, but remains on the vertical line through the fixed point. We propose to study the analogous case where the second end of the chain is fixed to a point vertically below the first point at a distance smaller than the length of the chain. The mathematical problem is *not* analogous to that of Kolodner; the different boundary condition changes its character. An accessory equation is introduced which makes a treatment along the lines of Kolodner's attack rather difficult if not impossible.

So we turn to a different method, which has been used extensively in recent times. It involves establishing the existence of continua of solution via topological degree methods. Thus one obtains global extensions of local branches given by the implicit function theorem. A very practical and well known approach is that inaugurated by Rabinowitz in his treatment of global continua of solutions to certain bifurcation problems. Unfortunately the problem at hand resists that approach as well. The natural setup for this method would be a C^2 space, but the trivial solution of the problem does not belong to it. This solution is described by the following $x(s) = (x_1(s), x_2(s), x_3(s))$:

$$x_1(s) = x_2(s) = 0$$

$$x_3(s) = \begin{cases} s, & s \in \left[0, \frac{l+a}{2}\right], \\ l + a - s, & s \in \left[\frac{l+a}{2}, l\right]. \end{cases}$$

We have used the coordinate system shown in Figure 1.

The function describing the trivial solution is not differentiable, so bifurcation from it cannot be established by the usual analytical tools. But a heuristic argument shows that for infinite angular velocity there must be a definite solution, in shape roughly similar to a catenary.

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