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# LYAPUNOV-TYPE FUNCTIONS FOR NTH ORDER LINEAR ORDINARY DIFFERENTIAL EQUATIONS

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1. **Introduction.** The purpose of this paper is to introduce and utilize what we shall refer to as *Lyapunov-type functions* for the $n$th order real linear ordinary differential equation

$$\sum_{i=0}^{n} p_i(t)y^{(i)} = 0, \qquad t \in J \tag{1.1}$$

where $J$ is the half-open interval $[\alpha, \infty)$, $p_i \in \mathscr{C}(J \rightarrow \mathbf{R})$ $(i = 0, \cdots, n)$ and $p_n(t) \neq 0$ for $t \in J$. We introduce the concept in § 1 and develop a *systematic procedure* for constructing Lyapunov-type functions in § 2. In § 3, we demonstrate how Lyapunov-type functions can be utilized to find coefficient criteria for (1.1) which guarantee that certain two-point boundary value problems are uniquely solvable. In a later paper, we apply Lyapunov-type functions to obtain oscillation results for higher order linear differential equations.

DEFINITIONS. Suppose the functions $a_{ij}(0 \leqq i,\ j \leqq n-1)$ and $b_i(0 \leqq i \leqq n-1)$ from $J$ to $\mathbf{R}$ are such that

$$\frac{d}{dt}\left[\sum_{i,j=0}^{n-1} a_{ij}(t)y^{(i)}(t)y^{(j)}(t)\right] = \sum_{i=0}^{n-1} b_i(t)(y^{(i)}(t))^2 \tag{1.2}$$

for all $t \in J$ whenever $y$ is a solution of (1.1). Then the function $\phi : \mathscr{C}^n(J \rightarrow \mathbf{R}) \rightarrow \mathscr{C}(J \rightarrow \mathbf{R})$ defined by

$$\phi(f) = \sum_{i,j=0}^{n-1} a_{ij}f^{(i)}f^{(j)} \tag{1.3}$$

is called a Lyapunov-type function for (1.1).

If $f \in C^n(J \rightarrow \mathbf{R})$ and $f_i \equiv f^{(i)}(i = 0, \cdots, n-1)$, then $\phi$ as defined in (1.3) can be regarded as a function from $J \times \mathbf{R}^n \rightarrow \mathbf{R}$ given by

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