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SYMMETRIC DERIVATIVES DEFINED BY WEIGHTED SPHERICAL MEANS

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ABSTRACT. We consider, for functions of several variables, symmetric derivatives defined by taking weighted spherical averages. We apply these derivatives to establish theorems of Lebesgue type for multiple trigonometric series.

1. **Introduction.** Let $f(t)$ be a function defined in a neighborhood of $t_0 \in \mathbf{R}$. We say f has a *first symmetric derivative* at t_0 with value s [9, vol. I, p. 59] if

$$\frac{1}{2}\{f(t_0 + t) - f(t_0 - t)\} = st + o(t) \tag{1.1}$$

as $t \to 0$. This definition has the following applications to formally integrated trigonometric series [9, vol. I, p. 322 and p. 324].

THEROEM A. *Let* $T: \sum_{n=-\infty}^{\infty} c_n e^{in\theta}$ *be a trigonometric series with* $c_n = O(1/n)$. *If* T *converges at* θ_0 *to finite sum* s *then*

$$f(\theta) = c_0\theta + \Sigma' \frac{c_n}{in} e^{in\theta} \tag{1.2}$$

has at θ_0 *a first symmetric derivative with value* s.

THEOREM B. *Suppose the coefficients of* $T: \Sigma\, c_n e^{in\theta}$ *satisfy* $c_n \to 0$ *as* $n \to \infty$. *If* T *converges at* θ_0 *to finite sum* s, *then the function* $f(\theta)$ *defined by* (1.2) *has at* θ_0 *a first symmetric approximate derivative equal to* s. *That is, the limit in* (1.1) *exists as it tends to* 0 *through a set having* 0 *as a point of density.*

A two dimensional version of (1.1) and of Theorems A and B appears in [5] and [6]. In two dimensions let us write $x = (x_1, x_2) = te^{i\theta}$ and $n = (n_1, n_2)$. Let

$$\Omega(\theta) = \cos\theta + \sin\theta. \tag{1.3}$$

Let $L(x)$ be defined in a neighborhood of $x_0 \in E_2$ and integrable over each circle $|x - x_0| = t$, for t small. We say $L(x)$ has at x_0 a *first generalized symmetric derivative* with value s if

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