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THE N-TH ORDER ELLIPTIC BOUNDARY PROBLEM FOR NONCOMPACT BOUNDARIES

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0. **Introduction.** In this paper we will discuss the boundary problem

(p) $u \in \mathfrak{H}_N(\Omega)$, $\langle a \rangle u = f$ on $\Omega \cup \Gamma$, $\langle b^j \rangle u = \varphi_j$ on Γ, $j = 1, \ldots, r$,

where Ω and Γ are chosen as the halfspace $\mathbf{R}^{n+1}_+$ and its boundary $\partial\mathbf{R}^{n+1}_+ = \mathbf{R}^n$, as the simplest domain with noncompact boundary, while $\langle a \rangle$ denotes an even order elliptic differential expression over $\Omega \cup \Gamma$, with C^∞-coefficients. The order of $\langle a \rangle$ is N, and there are $r = N/2$ boundary conditions on Γ, determined by differential expressions $\langle b^j \rangle$ over a neighbourhood of Γ, of order $N_j < N$. The $\langle b^j \rangle$ again have C^∞-coefficients, and the system $\langle a \rangle, \langle b^1 \rangle, \ldots, \langle b^r \rangle$ locally is assumed to be elliptic -or, in other words, the $\langle b^j \rangle$ satisfy the so-called Lopatinskij-Shapiro conditions, locally, at each point of Γ.

Conditions at infinity will have to be added, of course, and we generally assume that $f \in L^2(\Omega)$, $\varphi_j \in \mathfrak{H}_{N-N_j-1/2}(\Gamma)$, with the L^2-Sobolev spaces. In fact our assumptions will restrict enough to imply the generalized Sobolev estimates of Agmon-Douglis-Nirenberg **[1]**, and *F.* Browder **[4]**. However, since the domain and its boundary are non-conpact, these do not imply finiteness of the nullspace or even normal solvability of the problem.

Our result, below, just asserts this normal solvability of (p), replacing in its proof the apriori estimate by a Banach algebra technique, under the following assumptions on the coefficients.

$$\langle a \rangle = a(x, D), \quad \langle b^j \rangle = b^j(\vec{x}, D),$$
$$a(x, \xi) = \sum_{|\alpha| \leq N} a_\alpha(x)\xi^\alpha, \quad b^j(\vec{x}, \xi) = \sum_{|\alpha|+k \leq N_j} b^j_{k,\alpha}(\vec{x})\vec{\xi}^\alpha \xi_0^k,$$
$$(0.1) \quad x = (x_0, \ldots, x_n) = (x_0, \vec{x}), \quad D = (D_0, \ldots, D_n) = (D_0, \vec{D}),$$
$$\xi = (\xi_0, \ldots, \xi_n) = (\xi_0, \vec{\xi}), \quad \Omega = \{x : x_0 > 0\}, \quad \Gamma = \{x : x_0 = 0\}$$
$$\lim_{\rho>0,\, \rho\to\infty} a_\alpha(\rho x) = a_\alpha(\infty x), \text{ as } |x| = 1, x_0 \geqq 0,$$
$$\lim_{\rho>0,\, \rho\to\infty} b^{j(\beta)}_{k,\alpha}(\rho \vec{x}) = b^{j(\beta)}_{k,\alpha}(\infty \vec{x}), \text{ as } |\vec{x}| = 1,$$

where the convergence of the limits in the last row is uniform in x (or $\vec{x}$), and the limits $a_\alpha(\infty \cdot x)$, $b^{j(\beta)}_{k,\alpha}(\infty \cdot \vec{x})$ are continuous over the half sphere and its boundary, for all β.

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