

## SOME REMARKS ON THE ORIGINS OF THE THEORY OF FUNCTIONS OF A REAL VARIABLE AND OF THE DESCRIPTIVE SET THEORY<sup>1</sup>

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The descriptive set theory arose from the “theory of functions of a real variable” and this, in turn, had its roots in analysis—more particularly—in some erroneous statements of the calculus which appeared around the beginning of the XIXth century.

The fact that even the greatest analysts were led to false statements is not surprising in view of the way in which the calculus was being developed. It is well known that at the time of Newton and Leibniz the fundamental notions of calculus were not well defined and its development was rather dictated by its striking applications to physics which called the attention of their creators and was not, and could not be, rigorous from the logical point of view.

Nevertheless, the founders of the calculus and their immediate followers, like the Bernoullis, Euler, Lagrange etc., edified a wonderful building without taking care of its foundations. Generally speaking, their results were correct, which was certainly due to their infallible intuition.

However, this pleasant time came to an end. The reasonings, not well founded, even of the most celebrated mathematicians, started to lead to false statements.

May I mention two false theorems of Cauchy (theorems which played an important role in the creation of the Theory of functions of a real variable).

In his fundamental “Cours d’Analyse”, edited in 1821, Cauchy claims that:

1. The limit of a convergent sequence of continuous functions is continuous [7, p. 120].

2. If a function  $f$  of two variables is continuous relatively to each variable separately, then  $f$  is continuous (relatively to both variables simultaneously).

Five years later, N.H. Abel gave a counter-example to the first of these statements.

For a counterexample to the second statement, one had to wait longer. It seems (according to P. Dugac; many details contained in my paper are

<sup>1</sup>Lecture held on May 17, 1977, at the University of Kansas at the Conference in honour of Prof. N. Aronszajn on his 70th birthday.