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COMPARISON THEOREMS FOR INFINITE SYSTEMS OF DIFFERENTIAL-FUNCTIONAL EQUATIONS AND STRONGLY COUPLED INFINITE SYSTEMS OF FIRST ORDER PARTIAL DIFFERENTIAL EQUATIONS

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ABSTRACT. The first part of the paper deals with an infinite system of weakly coupled (this means that every equation contains derivatives of only one unknown function) differential-functional equations of the form

$u_x^i(x, y) = f^i(x, y, u(x, y), u_y^i(x, y))$ $(i = 1, 2, \ldots)$ in $W = \{(x, y): x_0 \leqq x < x + b,\ y = (y_1, \ldots, y_n) \in R^n\}$. Here $u = (u^1, u^2, \ldots): W \ni (x, y) \to u(x, y) = (u^1(x,y), u^2(x, y), \ldots) \in \ell^\infty$ is the unknown function, $u_y^i(x, y) = \mathrm{grad}_y u^i(x, y)$ and f^i is a functional of the map u.

For the problem with the initial conditions

$$u^i(a_i, y) = p^i(y) \qquad (i = 1, 2, \ldots)$$

following questions are discussed: error bounds for an approximate solution, uniqueness of the solution and its continuous dependence on the right hand sides of the system and on the initial functions.

The results of the first part are applied to similar questions concerning a strongly coupled infinite system of first order partial differential equations

$u_x^i(x, y) = g^i(x, y, u(x, y), u_{y_1}(x, y), u_y^i(x, y))$ $(i = 1, 2, \ldots)$, where $u_{y_1} = (u_{y_1}^1, u_{y_1}^2, \ldots)$. The derivatives $u_{y_1}^j (j = 1, 2, \ldots)$ are responsible for the system to be strongly coupled and consequently the functions u^j are supposed to belong to a special class of analytic functions with respect to the variable y_1. This class was first taken advantage of by K. Nickel [3] in the theory of strongly coupled parabolic systems of nonlinear second order differential equations. The analyticity of u^j with respect to y_1 is essential in questions treated in this paper. This is shown by a counter-example constructed by A. Pliś [5] in which for a strongly coupled system of two linear equations there is no uniqueness for the Cauchy problem in the class C^∞. The local uniqueness (and also existence) of the solution, which is analytic with respect to y and belongs to the class C^1 with respect to x, was proved by M. Nagumo [2] in the finite case of differential equations, but under more restrictive assumptions on analyticity of the right hand sides of the strongly coupled system with respect to all arguments but the variable x. The argument of the first part is similar to that used by A. Pliś for a finite and weakly coupled system of first order partial differential equations in [4] and by the author in [7] and [8], where an existence theorem for infinite systems of differential-functional equations is proved too.

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