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EMBEDDING NONCOMPACT MANIFOLDS

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0. **Introduction.** Let X and Y denote PL spaces; that is, locally compact, separable, metric spaces each of which possesses a piecewise linear structure. The map $f: X \to Y$ is k-connected provided $\pi_i(f) = \pi_i(M_f, X) = 0$ for $i \leqq k$ where M_f denotes the mapping cylinder of f. In [6] Hudson proves that if f is a map between a compact PL manifold M^m and a PL manifold Q^q, $f \mid \partial M$ is an embedding of ∂M into ∂Q and $q - m \geqq 3$, then f is homotopic rel ∂M to a PL embedding provided $\pi_i(f) = 0$ for $i \leqq 2m - q + 1$ and $\pi_i(Q) = 0$ for $i \leqq 3m - 2q + 3$. Theorem 4.2 extends this theorem to the case where M is noncompact with appropriate additional assumptions. The assumption that Q be $3m - 2q + 3$ connected in Hudson's Theorem was later shown to be unnecessary (see [5, Ch. 12]) using surgery techniques. The techniques of this paper, which are an extension of those of [6] and [12] require this connectivity. Using PL approximation techniques Berkowitz and Dancis [1] were able to prove a theorem similar to Theorem 4.2 in the 3/4 range which does not require connectivity of Q.

The term space shall always mean a locally compact, separable, metric space. A polyhedron is a compact PL space. A PL m-manifold is a PL space locally homeomorphic with euclidean m-space. A map f between spaces X and Y is proper provided $f^{-1}(C)$ is compact for each compact subset C of Y. All maps and homotopies are assumed to be proper unless stated otherwise. The symbol "$\simeq$" is read "is homotopic to". The symbol Λ denotes the halfline $[0, \infty)$ and a subspace of a PL space X which is homemorphic to Λ is called a ray in X. All deformation retractions are assumed to be strong deformation retractions in the sense of [8]. The symbol ∂ denotes boundary and the abbreviation int denotes interior.

Sections 1, 2, and 3 should provide a self-contained treatment of infinite engulfing and its relation to connectivity at infinity (c.f., Lemma 2.1 of [1]).

1. Proper Collapsing.

DEFINITION 1.1. There is an elementary collapse from the polyhedron P to the polyhedron Q, denoted $P \searrow eQ$, provided $P = Q \cup D$ where D

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