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ON SOLVING THE EQUATION Aut $(X) = G$

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ABSTRACT. Given a finite group G, are there at most a finite number of finite groups X such that Aut $(X) \approx G$? If so, how does one determine all of them?

The first question has an affirmative answer. In this paper we consider the second question when G is a finite nonabelian simple group or a natural extension of it, a dihedral group, a dicyclic group or a quasidihedral group.

1. **Introduction.** G. A. Miller, in 1900, considered the problem of finding all finite groups having S_3 as their group of automorphisms. He proved that $C_2 \times C_2$ and S_3 are the only such groups. He also determined all finite groups having S_4 as their group of automorphisms. The reader is referred to [10]. De Vries and De Miranda [13] have investigated groups, finite and infinite, with a small number of automorphisms. Heineken and Liebeck [6] and Hallett and Hirsch [4] have worked on similar problems. Finite groups with abelian automorphism groups have been studied by B. E. Earnley [2]. Baer [14] proved in 1955 that a torsion group whose automorphism group is finite must itself be finite. Alperin [15] in 1961 characterized finitely generated groups with finite automorphism groups. Recently D. J. S. Robinson [16] has studied the consequences for a group of the finiteness of its automorphism group.

In (3.1) we prove that given a finite group G there are at most a finite number of finite groups X such that Aut $(X) \approx G$. It is to be expected that the problem of finding all these groups in any given instance would be difficult in general. However, a knowledge of Schur multipliers of various groups and some results concerning the group of central automorphisms of a group combined with elementary group theoretical arguments enables one to solve the above problem in certain instances.

2.1. **Terminology.** All groups mentioned in this paper are finite. Suppose G is a group. The following notation will be used:

$|G|$ = The order of the group G.

$|G|_p$ = The order of a Sylow p-subgroup of G.

π = The set of all primes.

$\pi(G) = \{p \in \pi | p | \ |G|\}$.

(m, n) = The greatest common divisor of the integers m and n.

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