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EXACT SOLUTION OF COUPLED PAIRS OF DUAL TRIGONOMETRIC SERIES

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ABSTRACT. Exact solutions are obtained for coupled dual trigonometric series that arise in the study of contact problem of a circular inclusion as well as a set of symmetric curvilinear cracks. The coupled dual series are reduced to coupled integral equations. Simple identities of the kernel functions allow us to decouple these integral equations into two uncoupled singular integral equations. One of these integral equations has a Cauchy type of singularity and can be reduced to the air-foil equation. The other has a logarithmic singularity and is reduced to two Volterra equations.

1. **Introduction.** Dual series equations arise frequently for the solution of mixed boundary value problems in mathematical physics. A comprehensive collection of potential problems with their formal solution is given in Sneddon's monograph [1].

In this paper we derive coupled dual series for an elasticity problem of a circular inclusion, with interface friction, via bipotential Airy-stress functions. Consider a circular insert in a two dimensional infinite medium under uniform tension as shown in Figure 1. Due to lack of bond at interface there will be two distinct sets of regions, namely regions of contact and regions of separations. Mathematically, boundary conditions lead to coupled dual series. Over the regions of contact, stresses and displacements are continuous and over the separated regions stresses vanish. In such a problem not only the contact stresses are unknown but also the regions of receding contact.

In this paper we obtain an exact solution of the coupled dual series. The analysis is formal. The existence and uniqueness of dual and triple series, in a rigorous fashion, have only been recently studied by Kelman [2], [3]. There are, at present, no theorems available for coupled dual series formally solved in the present paper.

Employing the usual notation [4], the boundary conditions for the problem are:

$$u_r = u_r', \; u_\theta = u_\theta', \; \sigma_r = \sigma_r', \; \tau_{r\theta} = \tau'_{r\theta} \text{ for } 0 \leqq \theta < \eta \tag{1}$$

$$\sigma_r = \sigma_r' = 0, \; \tau_{r\theta} = \tau'_{r\theta} = 0 \quad \text{for } \eta < \theta \leqq \pi/2. \tag{2}$$

The primed quantities refer to the insert and the angle η is the unknown angle of receding contact. The general solution of two dimen-

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