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MATRIX OPERATORS ON ℓ^p

D. BORWEIN AND A. JAKIMOVSKI

Introduction. Suppose throughout that $A = (a_{nk})$ $(n, k = 0, 1, \cdots)$ is an infinite matrix of complex numbers, and that

$$p \geqq 1 \text{ and } \frac{1}{p} + \frac{1}{q} = 1.$$

Let ℓ^p be the normed linear space of all complex sequences $x = \{x_n\}$ $(n = 0, 1, \cdots)$ with finite norm $\|x\|_p$, where

$$\|x\|_p = \left(\sum_{n=0}^{\infty} |x_n|^p \right)^{1/p} \text{when } 1 \leqq p < \infty$$

and

$$\|x\|_\infty = \sup_{n \geqq 0} |x_n|.$$

Let $B(\ell^p)$ be the normed linear space of all bounded linear operators on ℓ^p into ℓ^p; so that $A \in B(\ell^p)$ if and only if, for every $x \in \ell^p$, $y_n = (Ax)_n = \sum_{k=0}^{\infty} a_{nk}x_k$ is defined for $n = 0, 1, \cdots$, and $y = \{y_n\} \in \ell^p$. The norm $\|A\|$ of a matrix A in $B(\ell^p)$ is given by

$$\|A\| = \sup_{\|x\|_p \leqq 1} \|Ax\|_p.$$

It is known (see [8, p. 164]) that, for $1 \leqq p < \infty$, every operator in $B(\ell^p)$ has a matrix representation. Matrices in $B(\ell^p)$ have been characterized in terms of their elements only for $p = 1, 2, \infty$. Crone [1] characterized matrices in $B(\ell^2)$ by means of rather complicated conditions that are difficult to apply. The following are characterizations of $B(\ell^1)$ and $B(\ell^\infty)$ (see [8, p. 167 and p. 174]): $A \in B(\ell^1)$ if and only if

$$(\mathrm{C}_1) \qquad \sup_{k \geqq 0} \sum_{n=0}^{\infty} |a_{nk}| < \infty.$$

$A \in B(\ell^\infty)$ if and only if

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