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CANTOR SETS IN 3-MANIFOLDS[1]

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1. **Introduction.** We answer the following 3-dimensional questions posed by Bing and Daverman which show that wild Cantor sets in 3-manifolds behave essentially like a 1-dimensional polyhedron. Furthermore, any compactum in the interior of a 3-manifold can be approximated by a Cantor set. The questions below are unsolved for $n > 3$; however, some partial solutions are known and pointed out.

QUESTION 1.1 (BING). [4, Question 1, p. 17]. What are necessary and sufficient conditions on an n-manifold M^n without boundary in order that it have the property that each Cantor set in M^n lies in an open n-cell in M^n?

If we stipulate that the M^n in Bing's question is closed, then an answer to Bing's question is: M^n is homeomorphic to the n-sphere for $n = 3$ [8] and $n > 4$ [13].

DEFINITION 1.1. A compactum K in an n-manifold M^n is said to be *approximable by Cantor sets* if for each neighborhood U of K there exists a Cantor set C in U such that a loop γ in $M^n - U$ is inessential in $M^n - K$ if and only if γ is inessential in $M^n - C$. We say that the Cantor set C *approximates* K *with respect to* U.

QUESTION 1.2 (DAVERMAN). Is every compactum in the interior of an n-manifold approximable by Cantor sets?

Recent work of Daverman and Edwards [7] has shown that the answer to Question 1.2 is affirmative if K is a closed, flat, PL $(n - 2)$-dimensional manifold.

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2. **Approximating compacta by Cantor sets.**

LEMMA 2.1. *Suppose P is a polyhedral finite graph in the interior of a 3-manifold M. Then P is approximable by Cantor sets.*

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[1]Some of this work is found in the author's doctoral thesis under Professor J. W. Cannon at the University of Wisconsin, Madison.