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A(X) AND GB-NOETHERIAN RINGS

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ABSTRACT. Three theorems concerning a Noetherian ring A are proved: (1) $A(X)$ is a GB-ring (that is, adjacent prime ideals in integral extension rings of A contract in A to adjacent prime ideals) if and only if $A[X]$ is a GB-ring; (2) if A is local and altitude $A = n + 2$ $(n \geqq 0)$, then $A(X_1, \cdots, X_n)$ is a GB-ring if and only if A satisfies the second chain condition for prime ideals (s.c.c.); and, (3) each GB-local domain A is such that $A(X)$ is a GB-ring if and only if each GB-local domain satisfies the s.c.c.

1. **Introduction.** All rings in this article are assumed to be commutative with identity, and the undefined terminology is the same as that in [**6**].

GB-rings were considered in their own right for the first time in [**11**], and therein a number of properties of such rings were proved. The reason such rings are of interest is that they are closely related to the (catenary) chain conjectures. Specifically, these conjectures are concerned with whether or not certain rings satisfy the chain condition for prime ideals (c.c.) (see (3.6.4) for the definition), and it is known [**11**, (3.8)] that a ring A satisfies the c.c. if and only if A is catenary and a GB-ring. Now, catenary rings have been deeply investigated in a number of papers, but, except for [**11**], GB-rings seem not to have been considered as an object of study in their own right. Even so, the literature does contain scattered information on GB-rings. For instance, it is easily seen that M. Nagata's example [**6**, Example 2, pp. 203–205] is not a GB-ring. And, probably the most important fact obtained on such rings is I. Kaplansky's 1972 paper [**3**] in which he gave a negative answer to the following question asked by W. Krull in 1937 in [**4**, p. 755]: is every integrally closed integral domain a GB-ring? (However, it is still an open problem if the integral closure of a Noetherian domain is necessarily a

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